PROBLEMS

Problem proposals and solutions should be sent to Bruce Shawyer, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. A1C 5S7. Proposals should be accompanied by a solution, together with references and other insights which are likely to be of help to the editor. When a proposal is submitted without a solution, the proposer must include sufficient information on why a solution is likely. An asterisk (⋆) after a number indicates that a problem was proposed without a solution.

In particular, original problems are solicited. However, other interesting problems may also be acceptable provided that they are not too well known, and references are given as to their provenance. Ordinarily, if the originator of a problem can be located, it should not be submitted without the originator's permission.

To facilitate their consideration, ***please send your proposals and solutions on signed and separate standard*** $8\frac{1}{2}$"×11" ***or*** A4 ***sheets of paper****. These may be typewritten or neatly hand-written, and should be mailed to the Editor-in-Chief, to arrive no later than* **1 March 2002**. *They may also be sent by email to* **crux-editors@cms.math.ca**. *(It would be appreciated if email proposals and solutions were written in LaTeX). Graphics files should be in LaTeX format, or encapsulated postscript. Solutions received after the above date will also be considered if there is sufficient time before the date of publication. Please note that we do not accept submissions sent by FAX.*

The name of Šefket Arslanagić, University of Sarajevo, Sarajevo, Bosnia and Herzegovina was omitted in error from the list of solvers of problems 2516 and 2522. Our apologies.

2651★. *Proposed by Murray S. Klamkin, University of Alberta, Edmonton, Alberta.* Dedicated to Professor M.V. Subbarao on the occasion of his 80th birthday. (Professor Klamkin offers a prize of $100 for the first correct solution received by the Editor-in-Chief.)

Let P be a non-exterior point of a regular n–dimensional simplex $A_0A_1A_2\dots A_n$ of edge length e. If

$$F = \sum_{k=0}^{n} PA_k + \min_{0\le k\le n} PA_k\,, \qquad F' = \sum_{k=0}^{n} PA_k + \max_{0\le k\le n} PA_k\,,$$

determine the maximum and minimum values of F and F'.

This problem was suggested by problem 2594 for a general triangle, and the proposer was trying to obtain a stronger inequality by finding the maximum of F.

2652★. *Proposed by Walther Janous, Ursulinengymnasium, Innsbruck, Austria.*

Let d, e and f be the sides of the triangle determined by the three points at which the internal angle-bisectors of given $\triangle ABC$ meet the opposite sides. Prove that

$$d^2 + e^2 + f^2 \;\leq\; \frac{s^2}{3}\,,$$

where s is the semiperimeter of $\triangle ABC$.

Show also that equality occurs if and only if the triangle is equilateral.

2653. *Proposed by Walther Janous, Ursulinengymnasium, Innsbruck, Austria.*

For whole numbers $n \geq 0$ and $N \geq 1$, evaluate the (combinatorial) sum

$$S_N(n) := \sum_{k \geq n} \binom{N}{2k}\binom{k}{n}\,.$$

2654 *Proposed by Christopher J. Bradley, Clifton College, Bristol, UK.*

Suppose that $\triangle ABC$ has medians AD, BE and CF. Suppose that L, M and N are points on the sides BC, CA and AB respectively.

Prove that the line through L parallel to AD, the line through M parallel to BE and the line through N parallel to CF are concurrent if and only if

$$\frac{BL^2}{BC^2} + \frac{CM^2}{CA^2} + \frac{AN^2}{AB^2} \;=\; \frac{LC^2}{BC^2} + \frac{MA^2}{CA^2} + \frac{NB^2}{AB^2}\,.$$

2655 *Proposed by Vedula N. Murty, Dover, PA, USA.*

Let a, b and c be the sides of $\triangle ABC$ and let s be its semiperimeter. Given that

$$\frac{bc}{b+c} + \frac{ca}{c+a} + \frac{ab}{a+b} \;=\; s\,,$$

show that $\triangle ABC$ is equilateral.

2656★. *Proposed by Vedula N. Murty, Dover, PA, USA.*

For positive real numbers a, b and c, show that

$$\frac{(1-b)(1-bc)}{b(1+a)} + \frac{(1-c)(1-ca)}{c(1+b)} + \frac{(1-a)(1-ab)}{a(1+c)} \;\geq\; 0\,.$$

2657. *Proposed by Aram Tangboondouangjit, Carnegie Mellon University, Pittsburgh, PA, USA.*

Prove that

$$\sum_{n=0}^{2k-1} \tan\left(\frac{(4n-1)\pi + (-1)^n 4\theta}{8k}\right) \;=\; \frac{2k}{1 + (-1)^{k+1}\sqrt{2}\sin\theta}\,.$$

2658. *Proposed by Juan-Bosco Romero Márquez, Universidad de Valladolid, Valladolid, Spain.*

Let $\triangle ABC$ have $\angle BCA = 90^\circ$. Squares $ACDE$ and $CBGF$ are drawn externally to the triangle. Suppose that AG and BE intersect at M. Show that M lies on the altitude CN.

2659. *Proposed by D.J. Smeenk, Zaltbommel, the Netherlands.*

In $\triangle ABC$, the side BC is fixed. A is a variable point. Assume that $AC > AB$. Let M be the mid-point of BC, let O be the circumcentre of $\triangle ABC$, let R be the circumradius, let G be the centroid and H the orthocentre. Assume that the Euler line, OH, is perpendicular to AM.

1. Determine the locus of A.
2. Determine the range of $\angle BGC$.

2560. . *Proposed by José Luis Díaz, Universitat Politècnica de Catalunya, Terrassa, Spain.*

Let $z_1, z_2, \ldots, z_n$ be distinct non-zero complex numbers. Prove that

$$\sum_{j=1}^{n} z_j^{n-1}\left(1+\prod_{\substack{k=1\\k\neq j}}^{n} z_k\right)\prod_{\substack{k=1\\k\neq j}}^{n}\frac{1}{z_k-z_j}$$

is a real number, and determine its value.

2561. *Proposed by Paul Yiu, Florida Atlantic University, Boca Raton, FL, USA.*

Let H be the orthocentre of acute-angled $\triangle ABC$ in which $\tan\left(\frac{A}{2}\right) = \frac{1}{2}$. Show that the sum of the radii of the incircles of $\triangle AHB$ and $\triangle AHC$ is equal to the inradius of $\triangle ABC$.

Is the converse true?

2562. *Proposed by Christopher J. Bradley, Clifton College, Bristol, UK.*

Suppose that $\triangle ABC$ is acute-angled, has inradius r and has area Δ. Prove that

$$\left(\sqrt{\cot A}+\sqrt{\cot B}+\sqrt{\cot C}\right)^2 \leq \frac{\Delta}{r^2}\,.$$

2563. *Proposed by Antreas P. Hatzipolakis, Athens, Greece; and Paul Yiu, Florida Atlantic University, Boca Raton, FL, USA.*

Suppose that the incircle of $\triangle ABC$ is tangent to the circle with BC as diameter. Show that the excircle on BC has radius equal to BC.