

PROBLEMS

Problem proposals and solutions should be sent to Bruce Shawyer, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. A1C 5S7. Proposals should be accompanied by a solution, together with references and other insights which are likely to be of help to the editor. When a proposal is submitted without a solution, the proposer must include sufficient information on why a solution is likely. An asterisk () after a number indicates that a problem was proposed without a solution.*

In particular, original problems are solicited. However, other interesting problems may also be acceptable provided that they are not too well known, and references are given as to their provenance. Ordinarily, if the originator of a problem can be located, it should not be submitted without the originator's permission.

*To facilitate their consideration, please send your proposals and solutions on signed and separate standard $8\frac{1}{2}'' \times 11''$ or A4 sheets of paper. These may be typewritten or neatly hand-written, and should be mailed to the Editor-in-Chief, to arrive no later than **1 October 2001**. They may also be sent by email to cru-x-editors@cms.math.ca. (It would be appreciated if email proposals and solutions were written in \LaTeX). Graphics files should be in *epic* format, or encapsulated postscript. Solutions received after the above date will also be considered if there is sufficient time before the date of publication. Please note that we do not accept submissions sent by FAX.*



In this special Murray Klamkin Birthday issue, we are pleased to present some problems posed by Murray, and some problems dedicated to him.

2613. *Proposed by Murray S. Klamkin, University of Alberta, Edmonton, Alberta. Dedicated to Toshio Seimiya, and based on his problem 2515.*

In $\triangle ABC$, the three cevians AD , BE and CF through a non-exterior point P are such that $AF + BD + CE = s$ (the semi-perimeter). Characterize $\triangle ABC$ for each of the cases when P is (i) the orthocentre, and (ii) the Lemoine point.

[Ed. The Lemoine point is also known as the symmedian point. See, for example, James R. Smart, *Modern Geometries*, 4th Edition, 1994, Brooks/Cole, California, USA. p. 161.]

2614 *Proposed by Murray S. Klamkin, University of Alberta, Edmonton, Alberta. Dedicated to Toshio Seimiya, and suggested by his problem 2514.*

In $\triangle ABC$, the two cevians through a non-exterior point P meet AC and AB at D and E respectively. Suppose that $AE = BD$ and $AD = CE$. Characterize $\triangle ABC$ for the cases when P is (i) the orthocentre, (ii) the centroid, and (iii) the Lemoine point.

2615. *Proposed by Murray S. Klamkin, University of Alberta, Edmonton, Alberta.*

Suppose that x_1, x_2, \dots, x_n , are non-negative numbers such that

$$\sum x_1^2 + \sum (x_1 x_2)^2 = \frac{n(n+1)}{2},$$

where the sums here and subsequently are symmetric over the subscripts 1, 2, \dots , n .

(a) Determine the maximum of $\sum x_1$.

(b)★ Prove or disprove that the minimum of $\sum x_1$ is $\sqrt{\frac{n(n+1)}{2}}$.

2616★. *Proposed by Murray S. Klamkin, University of Alberta, Edmonton, Alberta.*

The following are three known properties of parabolas:

1. The area of the parabolic segment upon any chord as base is equal to $\frac{4}{3}$ times the area of the triangle having the same base and height (the tangent at a vertex of the triangle is parallel to the chord). [Due to Archimedes.]
2. The area of the parabolic segment cut off by any chord is $\frac{2}{3}$ times the area of the triangle formed by the chord and the tangents at its extremities.
3. The area of a triangle formed by three tangents to a parabola is $\frac{1}{2}$ times the area of the triangle whose vertices are the points of contact.

Are there any other smooth curves having any one of the above properties?

2617. *Proposed by Murray S. Klamkin, University of Alberta, Edmonton, Alberta.*

A problem in one book was to prove that each edge of an isosceles tetrahedron is equally inclined to its opposite edge. A problem in another book was to prove that the three angles formed by the opposite edges of a tetrahedron cannot be equal unless they are at right angles.

1. Show that only the second result is valid.
2. Show that a tetrahedron which is both isosceles and orthocentric must be regular.

2618 Proposed by Murray S. Klamkin, University of Alberta, Edmonton, Alberta.

Determine a geometric problem whose solution is given by the positive solution of the equation

$$3x^2 \left(\frac{1}{\sqrt{4R^2 + x^2 - a^2}} + \frac{1}{\sqrt{4R^2 + x^2 - b^2}} + \frac{1}{\sqrt{4R^2 + x^2 - c^2}} \right) \\ = (\sqrt{4R^2 + x^2 - a^2} + \sqrt{4R^2 + x^2 - b^2} + \sqrt{4R^2 + x^2 - c^2} + a + b + c),$$

where a , b , c and R are the sides and circumradius of a given triangle ABC .

2619. Proposed by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario, dedicated to Murray S. Klamkin, on his 80th birthday.

For natural numbers n , define functions f and g by $f(n) = \left\lfloor \frac{n}{\lfloor \sqrt{n} \rfloor} \right\rfloor$ and $g(n) = \left\lceil \frac{n}{\lfloor \sqrt{n} \rfloor} \right\rceil$. Determine all possible values of $f(n) - g(n)$, and characterize all those n for which $f(n) = g(n)$. [See [2000 : 197], Q. 8.]

2620. Proposed by Bill Sands, University of Calgary, Calgary, Alberta, dedicated to Murray S. Klamkin, on his 80th birthday.

Three cards are handed to you. On each card are three non-negative real numbers, written one below the other, so that the sum of the numbers on each card is 1. You are permitted to put the three cards in any order you like, then write down the first number from the first card, the second number from the second card, and the third number from the third card. You add these three numbers together.

Prove that you can always arrange the three cards so that your sum lies in the interval $\left[\frac{1}{3}, \frac{2}{3}\right]$.

2621. Proposed by J. Chris Fisher, University of Regina, Regina, Saskatchewan, and Bruce Shawyer, Memorial University of Newfoundland, St. John's, Newfoundland, dedicated to Murray S. Klamkin, on his 80th birthday.

You are given:

- (a) fixed real numbers λ and μ in the open interval $(0, 1)$;
- (b) circle ABC with fixed chord AB , variable point C , and points L and M on BC and CA , respectively, such that $BL : LC = \lambda : (1 - \lambda)$ and $CM : MA = \mu : (1 - \mu)$;
- (c) P is the intersection of AL and BM .

Find the locus of P as C varies around the circle ABC . (If $\lambda = \mu = \frac{1}{2}$, it is known that the locus of P is a circle.)

2622. Proposed by Christopher J. Bradley, Clifton College, Bristol, UK.

Find the exact value of $\sum_{n=0}^{\infty} \frac{2^{n+1}}{(2n+1)\binom{2n}{n}}$.

2623★. Proposed by Faruk Zejnulahi and Šefket Arslanagić, University of Sarajevo, Sarajevo, Bosnia and Herzegovina.

Suppose that $x_1, x_2, \dots, x_n > 0$. Let $x_{n+1} = x_1, x_{n+2} = x_2$, etc. For $k = 0, 1, \dots, n-1$, let

$$S_k = \sum_{j=1}^n \left(\frac{\sum_{i=0}^k x_{j+i}}{k} \right).$$

Prove or disprove that $S_k \geq S_{k+1}$.

2624. Proposed by H.A. Shah Ali, Tehran, Iran.

Let n black objects and n white objects be placed on the circumference of a circle, and define any set of m consecutive objects from this cyclic sequence to be an m -chain.

- Prove that, for each natural number $k \leq n$, there exists at least one $2k$ -chain consisting of k black objects and k white objects.
- Prove that, for each natural number $k \leq \sqrt{2n+5} - 2$, there exist at least two such disjoint $2k$ -chains.

2625. Proposed by Hojoo Lee, student, Kwangwoon University, Kangwon-Do, South Korea.

If R denotes the circumradius of triangle ABC , prove that

$$18R^3 \geq (a^2 + b^2 + c^2)R + \sqrt{3}abc.$$

