27: No 1 FEBRUARY / FÉVRIER 2001

Published by:

Canadian Mathematical Society
Société mathématique du Canada
577 King Edward, POB/CP 450-A
Ottawa, ON K1N 6N5
Fax/Télec: 613 565 1539

SYNOPSIS

1 The Academy Corner: No. 38 *Bruce Shawyer*

Featuring some readers' solutions to problems from the Memorial University Undergraduate Mathematics Competition, held in March 2000; and the problems of the 2000 William Lowell Putnam Mathematical Competition, reprinted with permission of the Mathematical Association of America.

5 The Olympiad Corner: No. 211 *R.E. Woodrow*

Featuring the problems of the Ukrainian Mathematical Olympiad, March 1997; the problems of the two papers of the Tenth Irish Mathematical Olympiad 1997; the problems of the Hungary-Israel Bi-national Mathematics Competition, 1997; the problems of the 10^{th} Grade, 36^{th} Armenian National Olympiad in Mathematics; a nice alternate solution to a problem treated in 1999 [1998 : 69; 1999 : 201]; readers' solutions to problems of the 47^{th} Polish Mathematical Olympiad; and solutions from our readers to problems of the 10^{th} Nordic Mathematical Contest.

19 Book Reviews *Alan Law*

Confronting the Core Curriculum
by John A. Dossey
Magic Tricks, Card Shuffling and Dynamic Computer Memories
by S. Brent Morris

22 A Heron Difference

K.R.S. Sastry

Is it possible that the lengths of two sides of a primitive Heron triangle have a common factor? The triangle with sides 9, 65, 70 and area 252 shows that this is possible. But notice that the common factor 5 is a prime of the form $4\lambda + 1$. Surprisingly, however, such a common factor cannot be a prime of the form $4\lambda - 1$.

Heron's name is familiar to anyone who has used the formula

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}\,, \qquad s = \tfrac{1}{2}(a+b+c)$$

to calculate the area Δ of a triangle with sides a, b, c. In a **Heron** triangle a, b, c, and Δ are positive integers. A Heron triangle is called **primitive** when $\gcd(a, b, c) = 1$. If a Heron triangle contains a right angle, then it is called a **Pythagorean** triangle. It is easy to see that there is no primitive Pythagorean triangle, where two sides have a common factor. The situation for Heron triangles is different. The aim of this paper is to prove that if there is a common (prime) factor of two sides of a primitive Heron triangle, then it is always a prime of the form $4\lambda + 1$.

Read on!

Featuring the solutions to the problems of the Final Round of the British Columbia Colleges Junior High School Mathematics Contest, May 5, 2000.

During one dull psychology class, I mindlessly scribbled in my notes: $1 + 2 + 3 + 4 + 5 = 15$. It was nothing overly profound, but I thought it was rather interesting that the sum of the numbers from 1 to 5, inclusive, resulted in the digit 1 followed by the digit 5. Curious to see what other combinations exhibited similar properties, I fooled around with some numbers and soon found out that $2 + \ldots + 7 = 27$, and $4 + \ldots + 29 = 429$. Thus, this motivated the following problem.

*We say that an ordered pair of positive integers (a, b) with $a < b$ is **astonishing** if the sum of the integers from a to b, inclusive, is equal to the digits of a followed by the digits of b. Determine all astonishing ordered pairs.*

Surprisingly, there are many beautiful patterns that arise from this problem, and we shall describe them in this article. I was curious to see if there was an ordered pair (a, b) where b has exactly 1999 digits, and while attempting to solve this problem, I discovered some extremely neat results. Hopefully this article will illustrate that often the simplest of ideas can lead to surprising and extraordinary results. In my case, however, this was completely by accident.

Problem.
Let A, B, and C denote the angles of a triangle ABC. The following inequality is well known:

$$1 < \cos A + \cos B + \cos C \leq \frac{3}{2}. \qquad (1)$$

We present two solutions to the left-hand side of the inequality and three solutions to the right-hand side inequality and draw the attention of the students to a wrong proof usually given to prove the right-hand side inequality. One of the proofs presented for the left-hand side inequality is believed to be new.

This month's "free sample" is:

2601. *Proposed by Michel Bataille, Rouen, France.*

Sequences $\{u_n\}$ and $\{v_n\}$ are defined by $u_0 = 4$, $u_1 = 2$, and for all integers $n \geq 0$, $u_{n+2} = 8t^2 u_{n+1} + \left(t - \frac{1}{2}\right) u_n$, $v_n = u_{n+1} - u_n$. For which t is $\{v_n\}$ a non-constant geometric sequence?