26: No 7 NOVEMBER / NOVEMBRE 2000

Published by:

Canadian Mathematical Society
Société mathématique du Canada
577 King Edward, POB/CP 450-A
Ottawa, ON K1N 6N5
Fax/Télec: 613 565 1539

SYNOPSIS

Daryl Tingley

Question B12 of the 1999 Canadian Open Mathematics Challenge was:

Q: *Triangle ABC is any one of the set of triangles having base BC equal to a and height from A to BC equal to h, with $h < \frac{\sqrt{3}}{2}a$. P is a point inside the triangle such that the value of*

$$\angle PAB = \angle PBA = \angle PCB = \alpha\,.$$

Show that the measure of α is the same for every triangle in the set.

The problem is one implication of the following locus problem:

L: *Let BC be parallel to the line ℓ such that the distance from BC to ℓ is less than $\frac{\sqrt{3}}{2}$ of the length of BC. Let A be a variable point on ℓ. Find the locus of all points P inside $\triangle ABC$ such that*

$$\angle PAB = \angle PBA = \angle PCB. \qquad (*)$$

The two constraints: "the distance from BC to ℓ is less than $\frac{\sqrt{3}}{2}$ of the length of BC", and "P inside $\triangle ABC$" restrict the more general question:

G: *Let BC be parallel to the line ℓ. Let A be a variable point of ℓ. Find the locus of all points P that satisfy* $(*)$.

These are nice problems for computer investigation.

Read on!