PROBLEMS

Problem proposals and solutions should be sent to Bruce Shawyer, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. A1C 5S7. Proposals should be accompanied by a solution, together with references and other insights which are likely to be of help to the editor. When a proposal is submitted without a solution, the proposer must include sufficient information on why a solution is likely. An asterisk ($\star$) after a number indicates that a problem was proposed without a solution.

In particular, original problems are solicited. However, other interesting problems may also be acceptable provided that they are not too well known, and references are given as to their provenance. Ordinarily, if the originator of a problem can be located, it should not be submitted without the originator's permission.

To facilitate their consideration, please send your proposals and solutions on signed and separate standard $8\frac{1}{2}$"×11" or A4 sheets of paper. These may be typewritten or neatly hand-written, and should be mailed to the Editor-in-Chief, to arrive no later than **1 April 2001**. *They may also be sent by email to* **crux-editors@cms.math.ca**. *(It would be appreciated if email proposals and solutions were written in LaTeX). Graphics files should be in epic format, or encapsulated postscript. Solutions received after the above date will also be considered if there is sufficient time before the date of publication. Please note that we do not accept submissions sent by FAX.*

2576. *Proposed by Joaquín Gómez Rey, IES Luis Buñuel, Alcorcón, Spain.*

Characterize the numbers n such that $n!$ finishes (in base 2 notation) with exactly $n-1$ zeros.

2577. *Proposed by Joaquín Gómez Rey, IES Luis Buñuel, Alcorcón, Spain.*

Let a_1, a_2, ..., a_n ($n \geq 2$) be positive integers. Determine the values of n and k ($2 \leq k \leq n$) for which the following identity holds:

$$\gcd_{1\leq i_1<\ldots<i_k\leq n}\left(\mathrm{lcm}\{a_{i_1},\ldots,a_{i_k}\}\right) = \mathrm{lcm}_{1\leq i_1<\ldots<i_k\leq n}\left(\gcd\{a_{i_1},\ldots,a_{i_k}\}\right)$$

2578. *Proposed by Joaquín Gómez Rey, IES Luis Buñuel, Alcorcón, Spain.*

For each integer n, determine the hundreds and the units digits of the number $\dfrac{1+5^{2n+1}}{6}$.

2579. *Proposed by Paul Yiu, Florida Atlantic University, Boca Raton, FL, USA.*

The excircle on the side BC of triangle ABC touches AC and AB, respectively at Y_A and Z_A. Likewise, the one on CA touches BC and BA at X_B and Z_B, and the one on AB touches CA and CB at Y_C and X_C. Let A' be the intersection of Z_BX_B and X_CY_C, B' be that of X_CY_C and Y_AZ_A, and C' be that of Y_AZ_A and Z_BX_B. Show that AA', BB' and CC' are concurrent. What is the point of intersection of these three lines?

2580. *Proposed by Ho-joo Lee, student, Kwangwoon University, Seoul, South Korea.*

Suppose that a, b and c are positive real numbers. Prove that

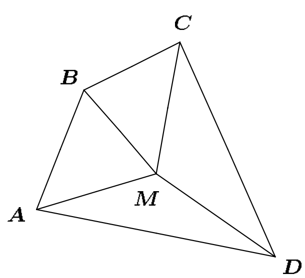
$$\frac{b+c}{a^2+bc}+\frac{c+a}{b^2+ac}+\frac{a+b}{c^2+ab} \;\le\; \frac{1}{a}+\frac{1}{b}+\frac{1}{c}\,.$$

2581. *Proposed by Ho-joo Lee, student, Kwangwoon University, Seoul, South Korea.*

Suppose that a, b and c are positive real numbers. Prove that

$$\frac{ab+c^2}{a+b}+\frac{bc+a^2}{b+c}+\frac{ca+b^2}{c+a} \;\ge\; a+b+c\,.$$

2583. *Proposed by Nairi M. Sedrakyan, Yerevan, Armenia.*



Given a point M inside the convex quadrangle (see diagram), such that $\angle AMB = \angle MAD + \angle MCD$, $\angle CMD = \angle MCB + \angle MAB$ and $MA = MC$.

Prove that $AB \cdot CM = BC \cdot MD$.

2584. *Proposed by Nairi M. Sedrakyan, Yerevan, Armenia.*

You are given that X, Y, Z and T are points on the chord AB of the circle Γ. Circles Γ_1 and Γ_2 pass through the points X and Y, and touch the circle Γ at points P and S, respectively, while the circles Γ_3 and Γ_4 pass through the points Z and T, respectively, and touch the circle Γ at points Q and R, respectively. Also, Q belongs to the arc APB and the segments XY and ZT do not have common points. Prove that the segments PR, QS and AB intersect at the same points.

2585. *Proposed by Vedula N. Murty, Visakhapatnam, India.*

Prove that, for $0 < \theta < \pi/2$,

$$\tan\theta + \sin\theta \;>\; 2\theta\,.$$

2586. *Proposed by Michael Lambrou, University of Crete, Crete, Greece.*

Find all (real or complex) solutions of the system

$$\begin{aligned} 3x + x^3 &= y(1 + 3x^2)\,, \\ 3y + y^3 &= z(1 + 3y^2)\,, \\ 3z + z^3 &= w(1 + 3z^2)\,, \\ 3w + w^3 &= x(1 + 3w^2)\,. \end{aligned}$$

2587. *Proposed by Peter Y. Woo, Biola University, La Mirada, CA, USA.*

In the half plane $Z = \{(x, y) : y \geq 0\}$, let f be the union of the set of all semicircles lying in Z with diameters on the x–axis, with the set of all lines in Z perpendicular to the x–axis.

Denote by F_{XY} the unique member of f that goes through any two points X and Y in Z. For any three points A, B and C in Z, denote by $\triangle ABC$ the curvilinear triangle formed by the arcs f_{AB}, f_{BC} and f_{CA}.

Let A, B and C be any three points on the x–axis. Let P be any point in the interior of $\triangle ABC$. Let $A' = f_{AP} \cap f_{BC}$, $B' = f_{BP} \cap f_{CA}$ and $C' = f_{CP} \cap f_{AB}$. Let α be the angle at A', interior to $\triangle CAA'$, let β be the angle at B' interior to $\triangle ABB'$, and let γ be the angle at C' interior to $\triangle BCC'$.

Prove that $\tan\left(\frac{\alpha}{2}\right)\tan\left(\frac{\beta}{2}\right)\tan\left(\frac{\gamma}{2}\right) = 1$.

2588. *Proposed by Niels Bejlegaard, Stavanger, Norway.*

Each positive whole integer a_k $(1 \leq k \leq n)$ is less than a given positive integer N. The least common multiple of any two of the numbers a_k is greater than N.

(a) Show that $\displaystyle\sum_{k=1}^{n} \frac{1}{a_k} < 2$.

(b)* Show that $\displaystyle\sum_{k=1}^{n} \frac{1}{a_k} < \frac{6}{5}$.

(c)* Find the smallest real number γ such that $\displaystyle\sum_{k=1}^{n} \frac{1}{a_k} < \gamma$.