**PROBLEMS**

Problem proposals and solutions should be sent to Bruce Shawyer, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. A1C 5S7. Proposals should be accompanied by a solution, together with references and other insights which are likely to be of help to the editor. When a submission is submitted without a solution, the proposer must include sufficient information on why a solution is likely. An asterisk (*) after a number indicates that a problem was submitted without a solution.

In particular, original problems are solicited. However, other interesting problems may also be acceptable provided that they are not too well known, and references are given as to their provenance. Ordinarily, if the originator of a problem can be located, it should not be submitted without the originator's permission.

To facilitate their consideration, please send your proposals and solutions on signed and separate standard 8.5" × 11" or A4 sheets of paper. These may be typewritten or neatly hand-written, and should be mailed to the Editor-in-Chief, to arrive no later than 1 April 2001. They may also be sent by email to crux-editors@cms.math.ca. (It would be appreciated if email proposals and solutions were written in \\text{LaTeX}.) Graphics files should be in \text{epic} format, or encapsulated postscript. Solutions received after the above date will also be considered if there is sufficient time before the date of publication. Please note that we do not accept submissions sent by FAX.

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2539. Correction. [The editor missed a subtle error in the proposer's proof. Thanks to Peter Y. Woo, Biola University, La Mirada, CA, USA for pointing out that the result could not be true.] Proposed by Ho-joo Lee, student, Kwangwoon University, Kangwon-Do, South Korea, adapted by the editor.

Let $ABCD$ be a convex quadrilateral with vertices oriented in the clockwise sense. Let $X$ and $Y$ be interior points on $AD$ and $BC$, respectively. Suppose that $P$ is a point between $X$ and $Y$ such that $\angle AXP = \angle BYP = \angle APB = \theta$ and $\angle CPD = \pi - \theta$ for some $\theta$.

(a) Prove that $AD \cdot BC \geq 4PX \cdot PY$.

(b) $*$ Find the case(s) of equality.

2563. Proposed by Nikolaos Dergiades, Thessaloniki, Greece.

You are given that angle $x$ satisfies the equation $a \sin x + b \cos x = c$.

(a) If $a$, $b$ and $c$ are real numbers, calculate angle $x$.

(b) Considering $a$, $b$ and $c$ as line segments, find a straight edge and compass construction for angle $x$. 
2564. Proposed by Darko Veljan, University of Zagreb, Zagreb, Croatia.

(a) Find all integer solutions \((a, b, c)\) of the equation \(\binom{a}{2} + \binom{b}{2} = \binom{c}{2}\), such that \(2 \leq a \leq b \leq c\).

(b) For each integer \(n \geq 1\), find at least one integer solution \((a, b, c)\) \((n \leq a \leq b \leq c)\) of the equation \(\binom{a}{n} + \binom{b}{n} = \binom{c}{n}\).

(c) For \(n = 3\), find at least one further solution for (b).

2565. Proposed by K.R.S. Sastry, Dodballapur, India.

A Heron triangle has integer sides and integer area. Show that there are exactly three pairs of Heron Triangles \(A_1B_1C_1\) and \(A_2B_2C_2\) such that 
\[B_1C_1 = B_2C_2, \quad A_1C_1 = A_2C_2, \quad \angle A_1B_1C_1 = \angle A_2B_2C_2\]
and \(A_2B_2 - A_1B_1 = 10\).

2566. Proposed by K.R.S. Sastry, Dodballapur, India.

Suppose that each of the three quadratics \(ax^2 + bx + c\), \(ax^2 + bx + (c+d)\) and \(ax^2 + bx + (c+2d)\) factors over the integers. Let \(S = ad > 0\). Show that \(S\) represents the area of some Pythagorean triangle (integer sided right triangle).

2567. Proposed by K.R.S. Sastry, Dodballapur, India.

In triangle \(ABC\), points \(P\) and \(Q\) are on the line segment \(BC\) such that \(AP\) and \(AQ\) are trisectors of \(\angle BAC\) and \(BQ = QC\). If \(AC = \sqrt{2}AQ\), find the measure of \(\angle BAC\).

2568. Proposed by K.R.S. Sastry, Dodballapur, India.

The sides \(a, b, c\) and \(d, e, f\) of a non-degenerate triangle \(ABC\) satisfy the relations \(b^2 = ca + a^2\) and \(c^2 = ab + b^2\). Find the measures of the angles of triangle \(ABC\).

2569. Proposed by Bill Sands, University of Calgary, Calgary, Alberta.

Suppose that \(a, b, c\) and \(d, e, f\) are real numbers satisfying

1. the pairwise sums of \(a, b, c\) are (in some order) \(d, e\) and \(f\); and

2. the pairwise products of \(d, e, f\) are (in some order) \(a, b\) and \(c\).

Find all possible values of \(a + b + c\).

2570. Proposed by Juan-Bosco Romero Márquez, Universidad de Valladolid, Valladolid, Spain.

Let \(C\) be a conic with focus \(F\) and directrix \(d\). Let \(A\) and \(B\) be the points of intersection of the conic with a line through the focus \(F\). Let \(I, J\) and \(K\) be the feet of the perpendiculars from \(A, F\) and \(B\) to \(d\), respectively.

Prove that the length of \(FJ\) is the harmonic mean of the lengths of \(AI\) and \(BK\).
2571. Proposed by Ho-joo Lee, student, Kwangwoon University, Seoul, South Korea.
Suppose that $a$, $b$ and $c$ are the sides of a triangle. Prove that
\[
\frac{1}{\sqrt[3]{a} + \sqrt[3]{b} - \sqrt[3]{c}} + \frac{1}{\sqrt[3]{b} + \sqrt[3]{c} - \sqrt[3]{a}} + \frac{1}{\sqrt[3]{c} + \sqrt[3]{a} - \sqrt[3]{b}} \geq \frac{3(\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c})}{a + b + c}.
\]

2572. Proposed by José Luis Díaz, Universitat Politècnica de Catalunya, Terrassa, Spain.
Let $a$, $b$, $c$ be positive real numbers. Prove that
\[
a^b b^c c^a \leq \left(\frac{a + b + c}{3}\right)^{a+b+c}.
\]
[Compare problem 2394 [1999 : 524], note by V.N. Murty on the generalization.]

2573. Proposed by Paul Yiu, Florida Atlantic University, Boca Raton, FL, USA.
Let $H$ be the orthocentre of triangle $ABC$. For a point $P$ not on the circumcircle of triangle $ABC$, denote by $X$, $Y$, $Z$ the reflections of $P$ in the sides $BC$, $CA$, and $AB$, respectively. Show that the areas of triangles $HYZ$, $HZX$, and $HXY$ are in constant proportions.

2574. Proposed by Paul Yiu, Florida Atlantic University, Boca Raton, FL, USA.
Let $P$ be a point in the interior of triangle $ABC$, whose centroid is $G$. Extend $AP$ to a point $X$ such that $PX$ is bisected by the line $BC$. Similarly, extend $BP$ to $Y$ and $CP$ to $Z$ such that $PY$ and $PZ$ are each bisected by $CA$ and $AB$, respectively. Show that the 6 points $A$, $B$, $C$, $X$, $Y$, $Z$, lie on a conic, and that the centre of the conic is the point $Q$ dividing $PG$ externally in the ratio $PQ : QG = 3 : -1$.

2575. Proposed by H. Fukagawa, Kani, Gifu, Japan.
Suppose that $\triangle ABC$ has a right angle at $C$. The circle, centre $A$ and radius $AC$ meets the hypotenuse $AB$ at $D$. In the region bounded by the arc $DC$ and the line segments $BC$ and $BD$, draw a square $EFGH$ of side $y$, where $E$ lies on arc $DC$, $F$ lies on $DB$ and $G$ and $H$ lie on $BC$. Assume that $BC$ is constant and that $AC = x$ is variable.
Find $\max y$ and the corresponding value of $x$. 

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