THE SKOLIAD CORNER

No. 46

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For your summer problem pleasure, we are giving the first rounds of two contests. I hope you enjoy the problems. My thanks go to Richard Nowakowski, Canadian Team Leader to the IMO in Buenos Aires for collecting them.

BUNDESWETTBEWERB MATHEMATIK 1997
Federal Contest in Mathematics (Germany)
First Round

1. Can you always choose 15 from 100 arbitrary integers so that the difference of any two of the chosen integers is divisible by 7?

What is the answer if 15 is replaced by 16? (Proof!)

2. Determine all primes $p$ for which the system

\[
p + 1 = 2x^2 \]
\[
p^2 + 1 = 2y^2 \]

has a solution in integers $x, y$.

3. A square $S_a$ is inscribed in an acute-angled triangle $ABC$ by placing two corners on the side $BC$ and one corner on $AC$ and $AB$, respectively. In a similar way, squares $S_b$ and $S_c$ are inscribed in $ABC$.

For which kind of triangle $ABC$ do the sides of $S_a, S_b$ and $S_c$ have equal length?

4. In a park there are 10 000 trees, placed in a square lattice of 100 rows and 100 columns. Determine the maximum number of trees that can be cut down satisfying the condition: sitting on a stump, you cannot see any other stump.

MACEDONIAN MATHEMATICAL COMPETITIONS 1997
Round 1 — Part 1

1. Find a number with three different digits, five times smaller than the sum of all the other numbers with the same digits. Determine all solutions!
2. Calculate \( \sqrt{7} - \sqrt{48} + \sqrt{5} - \sqrt{24} + \sqrt{3} - \sqrt{8} \).

3. Prove that, if \( a > 0, \ b > 0, \ c > 0 \), then
\[
\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}.
\]
Prove that the equality is valid if and only if \( a = b = c \).

4. Determine all the ordered pairs \((x, y)\), \(x \in \mathbb{N}, \ y \in \mathbb{N}\), for which three of the stated properties are true, and only one is false:
   (i) \( y \mid (x + 1) \),
   (ii) \( x = 2y + 5 \),
   (iii) \( 3 \mid (x + y) \),
   (iv) \( x + 7y \) is a prime number.

5. For which natural numbers \( n \) is the sum
\[
n^2 + (n + 1)^2 + (n + 2)^2 + (n + 3)^2
\]
divisible by 10?

6. Three tired travellers arrived at an inn and asked for a meal. The innkeeper did not have anything else to offer them but potatoes. While the potatoes were being baked the travellers fell asleep. After a while, when the potatoes were done, the first traveller woke up, ate \( \frac{1}{3} \) of the potatoes and went back to sleep without waking up the others. Then the second traveller woke up, ate \( \frac{1}{3} \) of the rest of the potatoes and went back to sleep. At last, the third traveller woke up, ate \( \frac{1}{3} \) of the rest of the potatoes and went back to sleep. The innkeeper watched all of these carefully and, in the end, when he counted the rest of the potatoes, there were \( 8 \). How many did the innkeeper bake?

Last issue we gave the problems of the Kangarou des Mathématiques, Épreuve EUROPEENNE Cadets, 1997. Here are the solutions.

1. c  2. c  3. c  4. b  5. b
6. c  7. b  8. e  9. e  10. a
11. d  12. b  13. c  14. c  15. c
16. c  17. a  18. d  19. c  20. b
21. b  22. b  23. d  24. c  25. d
26. e  27. b  28. a  29. e  30. e

That completes the Skoliad Corner for this number. Send me your comments and suggestions as well as suitable contest materials.