Problems proposals and solutions should be sent to Bruce Shawyer, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. Proposals should be accompanied by a solution, together with references and other insights which are likely to be of help to the editor. When a submission is submitted without a solution, the proposer must include sufficient information on why a solution is likely. An asterisk (*) after a number indicates that a problem was submitted without a solution.

In particular, original problems are solicited. However, other interesting problems may also be acceptable provided that they are not too well known, and references are given as to their provenance. Ordinarily, if the originator of a problem can be located, it should not be submitted without the originator's permission.

To facilitate their consideration, please send your proposals and solutions on signed and separate standard 8½" × 11" or A4 sheets of paper. These may be typewritten or neatly hand-written, and should be mailed to the Editor-in-Chief, to arrive no later than 1 October 2000. They may also be sent by email to crux-editors@cms.math.ca. (It would be appreciated if email proposals and solutions were written in LATEX. Graphics files should be in epic format, or encapsulated postscript). Solutions received after the above date will also be considered if there is sufficient time before the date of publication. Please note that we do not accept submissions sent by FAX.

The Editors would like to thank Waldemar Pompe for drawing to our attention that a venerable CRUX with MAYHEM problem proposer and solver will be celebrating his 90th birthday this month. So, we have seven Geometry problems in honour of Professor Toshio Seimiy'a's birthday. Happy birthday, and congratulations!!

2513. Proposed by Waldemar Pompe, University of Darmstadt, Darmstadt, Germany; dedicated to Prof. Toshio Seimiy'a on his 90th birthday.

A circle is tangent to the sides BC, AD of convex quadrilateral ABCD in points C, D, respectively. The same circle intersects the side AB in points K and L. The lines AC and BD meet in P. Let M be the mid-point of CD. Prove that if CL = DL, then the points K, P, M are collinear.

2514. Proposed by Toshio Seimiy'a, Kawasaki, Japan.

In ΔABC, the internal bisectors of ∠ABC and ∠BCA meet CA and AB at D and E respectively. Suppose that AE = BD and that AD = CE. Characterize ΔABC.

2515. Proposed by Toshio Seimiy'a, Kawasaki, Japan.

In ΔABC, the internal bisectors of ∠BAC, ∠ABC and ∠BCA meet BC, AC and AB at D, E and F respectively. Let p be the perimeter of ΔABC. Suppose that AF + BD + CE = ½p. Characterize ΔABC.
2516. Proposed by Toshio Seimiyah, Kawasaki, Japan.

In isosceles \( \triangle ABC \) (with \( AB = AC \)), let \( D \) and \( E \) be points on sides \( AB \) and \( AC \) respectively such that \( AD < AE \). Suppose that \( BE \) and \( CD \) meet at \( P \). Prove that \( AE + EP < AD + DP \).

2517. Proposed by Paul Yiu, Florida Atlantic University, Boca Raton, FL, USA.

Suppose that \( D, E, F \) are the mid-points of the sides \( BC, CA, AB \), respectively, of \( \triangle ABC \). Let \( P \) be any point in the plane of the triangle, distinct from \( A, B \) and \( C \).

1. Show that the lines parallel to \( AP, BP, CP \), through \( D, E, F \), respectively, are concurrent (at \( Q \), say).

2. If \( X, Y, Z \) are the symmetrics of \( P \) with respect to \( D, E, F \), respectively, show that \( AX, BY, CZ \) are concurrent at \( Q \).

2518. Proposed by Peter Y. Woo, Biola University, La Mirada, CA, USA.

If \( P \) is a point on the altitude \( AN \) of \( \triangle ABC \), if \( \angle PBA = 20^\circ \), if \( \angle PBC = 40^\circ \) and if \( \angle PCB = 30^\circ \), without using trigonometry, find \( \angle PCA \).

2519. Proposed by Peter Y. Woo, Biola University, La Mirada, CA, USA.

In \( \triangle ABC \), \( \angle ACB = 40^\circ \), \( AB \perp BC \), \( P \) and \( Q \) are points on \( AB \) and \( BC \) respectively with \( \angle PQB = 20^\circ \). Without using trigonometry, prove that \( AQ = 2BQ \) if and only if \( PQ = CQ \).

2520. Proposed by Paul Bracken, CRM, Université de Montréal, Montréal, Québec.

Let \( a, b \) be real numbers such that \( b < 0 \) and \( 1 + ax + bx^2 \geq 0 \) for every \( x \in [0, 1] \). Define

\[
F_n(a, b) = \int_0^1 (a + ax + bx^2)^n \, dx.
\]

Show that the following asymptotic expressions are valid for \( F_n(a, b) \) as \( n \to \infty \):

1. If \( a < 0 \) and \( b \leq 0 \), then

\[
F_n(a, b) = \frac{1}{an} + \frac{1}{n^2a} \left( 1 - \frac{2b}{a^2} \right) + O(n^{-3})
\]

2. If \( a \geq 0 \) and \( b < 0 \), then

\[
F_n(a, b) \sim \sqrt{\frac{\pi}{n b}} \left( 1 - \frac{a^2}{4b} \right)^{n+\frac{1}{2}}
\]
2521. Proposed by Eric Postpschil, Nashua, New Hampshire, USA.
Given a permutation \( \tau \), determine all pairs of permutations \( \alpha \) and \( \beta \), such that \( \tau = \beta \circ \alpha \) and \( \alpha^2 = \beta^2 = \iota \) (the identity permutation). That is, determine all factorizations of \( \tau \) into two permutations, each composed of disjoint transpositions.

2522*. Proposed by Walther Janous, Ursulinengymnasium, Innsbruck, Austria.
Suppose that \( a \), \( b \) and \( c \) are positive real numbers. Prove that
\[
\left( \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \left( \frac{1}{1 + a} + \frac{1}{1 + b} + \frac{1}{1 + c} \right) \geq \frac{9}{1 + abc}.
\]

2523. Proposed by Walther Janous, Ursulinengymnasium, Innsbruck, Austria.
Prove that, if \( t \geq 1 \), then
\[
\ln t \leq \frac{t - 1}{2(t + 1)} \left( 1 + \sqrt{\frac{2t^2 + 5t + 2}{t}} \right).
\]

Also, prove that, if \( 0 < t \leq 1 \), then
\[
\ln t \geq \frac{t - 1}{2(t + 1)} \left( 1 + \sqrt{\frac{2t^2 + 5t + 2}{t}} \right).
\]

2524. Proposed by Václav Konečný, Ferris State University, Big Rapids, MI, USA.
What conditions must the real numbers \( x \), \( y \) and \( z \) satisfy so that
\[
cot x \cot y \cot z = \cot x + \cot y + \cot z,
\]
where \( x, y, z \neq n\pi \), with \( n \) being an integer?

2525. Proposed by Václav Konečný, Ferris State University, Big Rapids, MI, USA.
Consider the recursions: \( x_{n+1} = 2x_n + 3y_n \), \( y_{n+1} = x_n + 2y_n \), with \( x_1 = 2, y_1 = 1 \). Show that, for each integer \( n \geq 1 \), there is a positive integer \( K_n \) such that
\[
x_{2n+1} = 2 \left( K_n^2 + (K_n + 1)^2 \right).
\]