THE SKOLIAD CORNER
No. 43

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We begin the first issue of the new Millennium with the problems from Part I of the Alberta High School Mathematics Competition written in November, 1999. Thanks go to Ted Lewis, University of Alberta, for forwarding this contest for use in the Corner.

THE ALBERTA HIGH SCHOOL MATHEMATICS COMPETITION
PART I
November 16, 1999

1. Subtracting 99% of 19 from 19% of 99, the difference \( d \) satisfies
   (a) \( d < -1 \)  (b) \( d = -1 \)  (c) \( -1 < d < 1 \)  (d) \( d = 1 \)  (e) \( d > 1 \)

2. Suppose you multiply three different positive prime numbers together and get a product which is greater than 1999. The smallest possible size of the largest of your primes is
   (a) 11  (b) 13  (c) 17  (d) 19  (e) undefined

3. Suppose you multiply three different positive prime numbers together and get a product which is greater than 1999. The largest possible size of the smallest of your primes is
   (a) 3  (b) 5  (c) 7  (d) 11  (e) undefined

4. The number of two-digit positive integers such that the difference between the integer and the product of its digits is 12 is
   (a) 1  (b) 2  (c) 3  (d) 4  (e) none of these

5. The non-zero slope of a certain straight line is equal to its \( y \)-intercept if and only if the \( x \)-intercept \( a \) satisfies
   (a) \( a = 1 \)  (b) \( a = -1 \)  (c) \( a > 0 \)  (d) \( a < 0 \)  (e) none of these

6. \( A \) and \( B \) are positive integers. The sum of the digits of \( A \) is 19. The sum of the digits of \( B \) is 99. The smallest possible sum of the digits of the number \( A + B \) is
   (a) 1  (b) 19  (c) 20  (d) 118  (e) none of these
7. O is the origin of the coordinate plane. A, B and C are points on the x-axis such that \(OA = AB = BC = 1\). D, E and F are points on the y-axis such that \(OD = DE = EF \geq 1\). If \(CD \cdot AF = BE^2\), then \(OD\) is

(a) 1 \hspace{1cm} (b) \(\sqrt{7}\) \hspace{1cm} (c) 7 \hspace{1cm} (d) 49 \hspace{1cm} (e) none of these

8. The integer closest to \(100(12 - \sqrt{143})\) is

(a) 2 \hspace{1cm} (b) 3 \hspace{1cm} (c) 4 \hspace{1cm} (d) 5 \hspace{1cm} (e) 6

9. A bag contains four balls numbered \(-2, -1, 1\) and \(2\). Two balls are drawn at random from the bag, and the numbers on them are multiplied together. The probability that this product is either odd or negative (or both) is

(a) \(\frac{1}{6}\) \hspace{1cm} (b) \(\frac{1}{2}\) \hspace{1cm} (c) \(\frac{9}{16}\) \hspace{1cm} (d) \(\frac{2}{3}\) \hspace{1cm} (e) \(\frac{3}{4}\)

10. The number of positive perfect cubes which divide \(9^9\) is

(a) 6 \hspace{1cm} (b) 9 \hspace{1cm} (c) 18 \hspace{1cm} (d) 27 \hspace{1cm} (e) none of these

11. In the quadratic equation \(x^2 - 14x + k = 0\), \(k\) is a positive integer. The roots of the equation are two different prime numbers \(p\) and \(q\). The value of \(\frac{p + q}{p}\) is

(a) 2 \hspace{1cm} (b) \(\frac{106}{45}\) \hspace{1cm} (c) \(\frac{130}{33}\) \hspace{1cm} (d) \(\frac{170}{13}\) \hspace{1cm} (e) none of these

12. In the quadrilateral \(ABCD\), \(AB\) is parallel to \(CD\), \(AB = 4\) and \(BC = CD = 9\). \(X\) is on \(BC\) and \(Y\) is on \(DA\) such that \(XY\) is parallel to \(AB\). If the quadrilaterals \(ABXY\) and \(YXCD\) are similar, distance \(BX\) is

(a) 3 \hspace{1cm} (b) 3.6 \hspace{1cm} (c) 5.4 \hspace{1cm} (d) 6 \hspace{1cm} (e) none of these

13. The country of Magyaria has three kinds of coins, each worth a different integral number of dollars. Matthew collected four Magyarian coins with a total worth of 28 dollars, while Daniel collected five with a total worth of 21 dollars. Each had at least one Magyarian coin of each kind. In dollars, the total worth of the three kinds of Magyarian coins is

(a) 16 \hspace{1cm} (b) 17 \hspace{1cm} (c) 18 \hspace{1cm} (d) 19 \hspace{1cm} (e) none of these

14. Colin wants a function \(f\) which satisfies \(f(f(x)) = f(x + 2) - 3\) for all integers \(x\). If he chooses \(f(1)\) to be 4 and \(f(4)\) to be 3, then he must choose \(f(5)\) to be

(a) 3 \hspace{1cm} (b) 6 \hspace{1cm} (c) 9 \hspace{1cm} (d) 12 \hspace{1cm} (e) 15

15. Lindsay summed all the integers from \(a\) to \(b\), including \(a\) and \(b\). She chose these numbers so that \(1 \leq a \leq 10\) and \(11 \leq b \leq 20\). This sum cannot be equal to

(a) 91 \hspace{1cm} (b) 92 \hspace{1cm} (c) 95 \hspace{1cm} (d) 98 \hspace{1cm} (e) 99
16. A set of points in the plane is such that each of the numbers 1, 2, 4, 8, 16 and 32 is a distance between two of the points in the set. The minimum number of points in this set is

(a) 4   (b) 5   (c) 6   (d) 7   (e) more than 7

Last issue we gave the thirty problems of Maxi éliminatoire 1996 of the 21ème Olympiade Belge organized by the Belgian Mathematics Teachers’ Association. Thanks go to Ravi Vakil for collecting this set when he was Canadian Team Deputy Leader to the International Mathematical Olympiad at Mumbai, India. Here are the answers.

1. c  2. Six  3. b  4. c  5. 243
6. c  7. d  8. c  9. 72m  10. c
16. c  17. a  18. d  19. d  20. b
26. c  27. Fourteen  28. e  29. e  30. a

Editor’s note: One arrives at 29. (e) by elimination, (a) is not allowed because of $x = 0$, none of (b), (c) (d) work because of $x = \pi$, since the expression is not identically zero. This leaves (e).

Articles

Readers will have noticed a lack of articles in recent issues. There are two reasons for this:

1. a lack of submitted articles;
2. most of those submitted have been at too high a level.

We do want to publish a CRUX article in each issue. So we warmly invite submissions. But please read the mandate of CRUX with MAYHEM, which is printed on the inside back cover:

**Crux Mathematicorum with Mathematical Mayhem** is a problem-solving journal at the senior secondary and university undergraduate levels for those who practice or teach mathematics.

So, please keep the level appropriate.

We welcome the new Articles Editor, Bruce Gilligan. Please send him your articles now.

Also, at this time we welcome Iliya Bluskov to the Board as a Problems Editor. Iliya has been a participant to CRUX since his student days, and has also helped out with editing problems for the past two years.