

# THE ACADEMY CORNER

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## A Trial Balloon

Vedula N. Murty

The following problem on heights and distances was set in the I.I.T. Entrance Exam in 1979.

*A balloon is observed simultaneously from three points, A, B and C, on a straight road directly beneath it. The angular elevation at B is twice that at A, and the angular elevation at C is three times that at A. If the distance between A and B is a, and the distance between B and C is b, find the height of the balloon in terms of a and b.*

Subsequently, this problem appeared in many textbooks in India with a solution which is straightforward. The height of the balloon is

$$y = \frac{a\sqrt{3b^2 + 2ab - a^2}}{2b}.$$

The conditions on  $a$  and  $b$  under which this solution is valid are not given in any of the solutions printed in the textbooks.

Professor M. Perisastry, a retired Professor of Mathematics at M.R. College, Viziahagaram, Andhra Pradesh, India, noted that

$$\begin{aligned} y > 0 &\implies 3b^2 + 2ab - a^2 > 0 \\ &\implies 4b^2 - (a - b)^2 > 0 \\ &\implies |a - b| < 2b \\ &\implies 0 < a < 3b. \end{aligned}$$

Moreover, it is easily seen that

$$\frac{a}{b} = \frac{\sin(3\alpha)}{\sin \alpha} = 3 - 4 \sin^2 \alpha,$$

where  $\alpha$  is the angle of elevation at  $A$ .

Since the angle of elevation at  $C$  is  $3\alpha < \pi/2$ , this implies that

$$\begin{aligned} 0 < \alpha < \frac{\pi}{6} &\implies 0 < \sin \alpha < \frac{1}{2} \\ &\implies \frac{a}{b} > 2 \\ &\implies a > 2b. \end{aligned}$$

Hence, 
$$\frac{a}{3} < b < \frac{a}{2}.$$

Readers of **CRUX with MAYHEM** may be interested in the above problem, and teachers should pay attention to the conditions under which a given solution is valid.

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### BAD CANCELLATIONS

$$\left| \frac{\tan^{-1}(2^{n+1})}{\tan^{-1}(2^n)} \cdot \frac{\cancel{n}^{3/2}}{(n+1)^{3/2}} \right| = 2$$

### QUESTIONS on MATHEMATICIANS

The year 1796 was the turning point in a (future) mathematician's career. Who was he?

And which mathematician was born in 1796?

What do Wilhelm Ackermann, Pavel Sergeevich Aleksandrov, Lester R. Ford, Ronald Martin Foster, Valeriĭ Ivanovich Glivenko, Kazimierz Kurtowski, and Carl Ludwig Seigel have in common?

What is the difference between Gustav Magnus Mittag-Leffler and Gaspard Monge?