As a contest this issue we give the Junior High School Mathematics Contest, Preliminary Round 1998 of the British Columbia Colleges which was written March 11, 1998. My thanks go to the contest organizer, Jim Totten, the University College of the Cariboo, for forwarding the 1998 contest materials to me. Students are given 45 minutes to respond to the 15 questions.

BRITISH COLUMBIA COLLEGES
Junior High School Mathematics Contest
Preliminary Round 1998
Time: 45 minutes

1. A number is prime if it is greater than one and divisible only by one and itself. The sum of the prime divisors of 1998 is:
   (a) 5   (b) 14   (c) 42   (d) 66   (e) 122

2. Successive discounts of 10% and 20% are equivalent to a single discount of:
   (a) 15%   (b) 25%   (c) 28%   (d) 30%   (e) 32%

3. Suppose that \( A = A^2 \) and \( A \Box B = A - 2B \). Then the value of \( 7 \Box 3 \) is:
   (a) 1   (b) 16   (c) 961   (d) 43   (e) 31

4. The expression that is not equal to the value of the four other expressions listed is:
   (a) \( 1 + 9 - 8 \)   (b) \( (1 + 9) \div (-\sqrt{9} + 8) \)   (c) \(-1 \times 9 + \sqrt{9} + 8 \)
   (d) \( (1 - \sqrt{9}) \times (9 - 8) \)   (e) \( 19 - 9 - 8 \)

5. The sum of all of the digits of the number \( 10^{75} - 75 \) is:
   (a) 8   (b) 655   (c) 664   (d) 673   (e) 675
6. A circle is divided into three equal parts and one part is shaded as in the accompanying diagram. The ratio of the perimeter of the shaded region, including the two radii, to the circumference of the circle is:

(a) 1  (b) \(\frac{2}{3}\)  (c) \(\frac{1}{3}\)  (d) \(\frac{3+\pi}{3\pi}\)  (e) \(\frac{\pi}{3}\)

7. The value of
\[
\frac{1}{2 - \frac{1}{\frac{3 - 1}{2 - \frac{1}{2}}}}
\]
is:
(a) \(\frac{3}{4}\)  (b) \(\frac{4}{5}\)  (c) \(\frac{5}{6}\)  (d) \(\frac{6}{7}\)  (e) \(\frac{6}{5}\)

8. If each small square in the accompanying grid is one square centimetre, then the area in square centimetres of the polygon \(ABCD\) is:

(a) 38  (b) 39  (c) 42  (d) 44  (e) 46

9. A point \(P\) is inside a square \(ABCD\) whose side length is 16. \(P\) is equidistant from two adjacent vertices, \(A\) and \(B\), and the side \(CD\) opposite these vertices. The distance \(PA\) equals:
(a) 8.5  (b) \(6\sqrt{3}\)  (c) 12  (d) 8  (e) 10

10. A group of 20 students has an average mass of 86 kg per person. It is known that 9 people from this group have an average mass of 75 kg
per person. The average mass in kilograms per person of the remaining 11 people is:

(a) 94       (b) 95       (c) 96       (d) 97       (e) none of these

11. In the following display each letter represents a digit:

```
 3 | B | C | D | E | 8 | G | H | I
```

The sum of any three successive digits is 18. The value of \( H \) is:

(a) 3       (b) 4       (c) 5       (d) 7       (e) 8

12. In the accompanying diagram \( \angle ADE = 140^\circ \). The sides are congruent as indicated. The measure of \( \angle EAD \) is:

(a) 30°       (b) 25°       (c) 20°       (d) 15°       (e) 10°

13. The area (in square units) of the triangle bounded by the \( x \)-axis and the lines with equations \( y = 2x + 4 \) and \( y = -\frac{2}{3}x + 4 \) is:

(a) 8       (b) 12       (c) 15       (d) 16       (e) 32

14. Two diagonals of a regular octagon are shown in the accompanying diagram. The total number of diagonals possible in a regular octagon is:

(a) 8       (b) 12       (c) 16       (d) 20       (e) 28
15. A local baseball league is running a contest to raise money to send a team to the provincial championship. To win the contest it is necessary to determine the number of baseballs stacked in the form of a rectangular pyramid. The fifth and sixth levels from the base of the stack of baseballs are shown. If the stack contains a total of seven levels, the number of baseballs in the stack is:

(a) 100  (b) 112  (c) 166  (d) 168  (e) 240

In the May number of the Corner we gave the problems of the 15th W.J. Blundon Contest written by students in Newfoundland and Labrador. Next we give the "official" solutions. My thanks go to Bruce Shawyer for forwarding the contest and solutions to me.

15th W.J. BLUNDON CONTEST
February 18, 1998

1. (a) Find the exact value of

$$\frac{1}{\log_2 36} + \frac{1}{\log_3 36}.$$  

Solution. \(\frac{1}{\log_2 36} + \frac{1}{\log_3 36} = \log_{36} 2 + \log_{36} 3 = \log_{36} 6 = \frac{1}{2}.$$  

(b) If \(\log_{15} 5 = a,\) find \(\log_{15} 9\) in terms of \(a.\)

Solution. \(1 = \log_{15} 15 = \log_{15} (5 \cdot 3) = \log_{15} 5 + \log_{15} 3 = a + \log_{15} 3.\)

\(\log_{15} 3 = 1 - a \Rightarrow \log_{15} 9 = \log_{15} 3^2 = 2 \log_{15} 3 = 2(1 - a).\)

2. (a) If the radius of a right circular cylinder is increased by 50\% and the height is decreased by 20\%, what is the change in the volume?

Solution. \(V_2 = \pi (1.5r)^2 (0.8h) = 1.8(\pi r^2h) = 1.8V_1.\) So the volume is increased by 80\%.

(b) How many digits are there in the number \(2^{1998} \cdot 5^{1988}\, ?\)

Solution. \(2^{1998} \cdot 5^{1988} = 2^{10} \cdot 2^{1988} \cdot 5^{1988} = 1024 \cdot 10^{1988},\) which has 1988 + 4 = 1992 digits.

3. Solve: \(3^2 + x + 3^{2-x} = 82.\)
Solution.

\[
3^{2+x} + 3^{2-x} = 82
\]
\[
9 \cdot 3^x + \frac{9}{3^x} = 82
\]
\[
9(3^x)^2 - 82(3^x) + 9 = 0
\]
\[
(9 \cdot 3^x - 1)(3^x - 9) = 0
\]
\[
x = -2 \quad x = 2
\]

4. Find all ordered pairs of integers such that \(x^6 = y^2 + 53\).

**Solution.**

\[
x^6 = y^2 + 53
\]
\[
x^6 - y^2 = 53
\]
\[
(x^3 - y)(x^3 + y) = 53
\]
\[
x^3 - y = 53
\]
\[
x^3 + y = 1
\]
\[
x^3 - y = 1
\]
\[
x^3 + y = 53
\]
\[
x^3 - y = -53
\]
\[
x^3 + y = -1
\]
\[
x^3 - y = -1
\]
\[
x^3 + y = -53
\]
\[
x = 3
\]
\[
x = 3
\]
\[
x = -3
\]
\[
x = -3
\]
\[
y = -26
\]
\[
y = 26
\]
\[
y = 26
\]
\[
y = -26
\]

The pairs are \((3, -26), (3, 26), (-3, 26), (-3, -26)\).

5. When one-fifth of the adults left a neighbourhood picnic, the ratio of adults to children was \(2 : 3\). Later, when 44 children left, the ratio of children to adults was \(2 : 5\). How many people remained at the picnic?

**Solution.** Let \(A\) be the number of adults and \(C\) be the number of children initially at the picnic. After one-fifth of the adults left, four-fifths remain. So

\[
\frac{\frac{4}{5}A}{C} = \frac{2}{3} \implies 6A = 5C.
\]

After 44 children left

\[
\frac{C - 44}{\frac{4}{5}A} = \frac{2}{5} \implies 8A = 25C - 1100.
\]

Solving the two equations gives \(A = 50, C = 60\). The number remaining is then

\[
\frac{4}{5}(50) + (60 - 44) = 40 + 16 = 56.
\]
6. Find the area of a rhombus for which one side has length 10 and the diagonals differ by 4.

Solution.

\[(b + 2)^2 + b^2 = 100\]
\[2b^2 + 4b - 96 = 0\]
\[b^2 + 2b - 48 = 0\]
\[(b - 6)(b + 8) = 0\]
\[b = 6, \quad b \neq -8\]

Since the area of a rhombus is one half the product of the diagonals we get

\[A = \frac{1}{2}(2b)(2b + 4) = \frac{1}{2}(12)(16) = 96.\]

7. In how many ways can 10 dollars be changed into dimes and quarters, with at least one of each coin being used?

Solution. Let \(q\) be the number of quarters and \(d\) be the number of dimes. Then

\[25q + 10d = 1000\]
\[d = 100 - \frac{5}{2}q.\]

Since \(d\) must be an integer, \(q\) must be even. Also \(d\) must be positive. So

\[100 - \frac{2}{5}q > 0\]
\[q < 40.\]

So \(q\) must be an even positive integer less than 40, of which there are 19.
8. Solve: \( \sqrt{x+10} + \sqrt{x+10} = 12. \)

**Solution.** Let \( y = \sqrt{x+10} \). Then \( y^2 = \sqrt{x+10} \), and the equation becomes

\[
\begin{align*}
y^2 + y &= 12 \\
y^2 + y - 12 &= 0 \\
(y + 4)(y - 3) &= 0 \\
y &= -4, y = 3
\end{align*}
\]

9. Find the remainder when the polynomial \( x^{135} + x^{125} - x^{115} + x^5 + 1 \) is divided by the polynomial \( x^3 - x \).

**Solution.**

\[
x^{135} + x^{125} - x^{115} + x^5 + 1 = (x^3 - x)Q(x) + ax^2 + bx + c
\]

This must be valid for all values of \( x \). Substituting in \( x = 0, x = 1, \) and \( x = -1 \) gives:

\[
\begin{align*}
x = 0 : & \quad 1 = 0 + c \quad \implies \quad c = 1 \\
x = 1 : & \quad 3 = 0 + a + b + c \quad \implies \quad a + b = 2 \\
x = -1 : & \quad -1 = 0 + a - b + c \quad \implies \quad a - b = -2
\end{align*}
\]

Solving the system

\[
\begin{align*}
a + b &= 2 \\
a - b &= -2
\end{align*}
\]

gives \( a = 0, b = 2. \) So the remainder is \( 2x + 1. \)

10. Quadrilateral \( ABCD \) below has the following properties: (1) The midpoint \( O \) of side \( AB \) is the centre of a semicircle; (2) sides \( AD, DC \) and \( CB \) are tangent to this semicircle. Prove that \( AB^2 = 4AD \times BC. \)

**Solution.** First join the obvious lines in the given figure:

By the properties of tangents, \( DE = DF \) and \( CF = CG. \) Therefore \( \angle EDO = \angle FDO = \phi \) and \( \angle FCO = \angle GCO = \psi. \) Since \( OA = OB, \) we have \( \angle EAO = \angle GBO = \theta. \)
Summing the angles of quadrilateral $ABCD$, we get $\theta + 2\phi + 2\psi + \theta = 360^\circ$. Hence $\theta + \phi + \psi = 180^\circ$; that is, they are the angles of a triangle.

Considering triangles $AOD$, $DOC$ and $COB$, we get $\angle AOD = \psi$, $\angle DOC = \theta$ and $\angle COB = \phi$. Thus the three triangles are similar.

Considering the triangles $ADO$ and $BOC$, we have $\frac{AD}{AO} = \frac{OB}{BC}$, or $AD \times BC = AO \times OB$.

Since $AO = OB = \frac{1}{2} AB$, we get the result.

Last issue we gave the problems of the U.K. Intermediate Mathematical Challenge. Here are the solutions.

1. $C$  
2. $B$  
3. $A$  
4. $A$  
5. $E$

6. $C$  
7. $C$  
8. $A$  
9. $C$  
10. $B$

11. $C$  
12. $E$  
13. $D$  
14. $D$  
15. $D$

16. $D$  
17. $D$  
18. $C$  
19. $A$  
20. $E$

21. $E$  
22. $D$  
23. $A$  
24. $A$  
25. $B$

That completes the Skoliad Corner for this number. Send me your comments, suggestions, and most importantly, suitable contest materials for use in future issues.

Advance Announcement

The 1999 Summer Meeting of the Canadian Mathematical Society will take place at Memorial University in St. John's, Newfoundland, from Saturday, 29 May 1999 to Tuesday, 1 June 1999.

The Special Session on Mathematics Education will feature the topic What Mathematics Competitions do for Mathematics.

The invited speakers are

- Ed Barbeau (University of Toronto),
- Ron Dunkley (University of Waterloo),
- Tony Gardiner (University of Birmingham, UK), and
- Rita Janes (Newfoundland and Labrador Senior Mathematics League).

Requests for further information, or to speak in this session, as well as suggestions for further speakers, should be sent to the session organizers:

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