

THE ACADEMY CORNER

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Answer as many questions as you can. Complete solutions carry more credit than scattered comments about many problems.

- Determine whether or not the following system has any real solutions. If so, state how many real solutions exist.

$$x + \frac{1}{x} = y, \quad y + \frac{1}{y} = z, \quad z + \frac{1}{z} = x.$$

- The surface area of a closed cylinder is twice the volume. Determine the radius and height of the cylinder given that the radius and height are both integers.
- Prove that

$$1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2} < 2.$$

- Describe the set of points (x, y) in the plane for which

$$\sin(x + y) = \sin x + \sin y.$$

- In a parallelogram $ABCD$, the bisector of angle ABC intersects AD at P . If $PD = 5$, $BP = 6$ and $CP = 6$, find AB .
- Show that, where $k + n \leq m$,

$$\sum_{i=0}^n \binom{n}{i} \binom{m}{k+i} = \binom{m+n}{n+k}.$$

Send me your nice solutions!