

# THE SKOLIAD CORNER

No. 16

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As a contest this issue, we give the Saskatchewan Senior Mathematics Contest. This contest was written Wednesday, February 22, 1995. My thanks go to Garreth Griffith, Mathematics Department, The University of Saskatchewan, long time organizer of the contests in Saskatchewan, for permission to use the contest and for the solutions we shall give next issue.

## 1995 SASKATCHEWAN SENIOR MATHEMATICS CONTEST

February 22, 1995 — Time: 1.5 hours

**1.** They sell regular and jumbo sized orders of fish at Jerry's Fish & Chips Emporium. The jumbo order costs  $(\frac{4}{3})$  times as much as a regular one and an order of chips costs \$1.30.

Last Thursday, the Emporium was quite busy over the lunch break (11:30 am - 1:30 pm). Exactly thirteen jumbo sized orders of fish along with a quantity of regular orders of fish and chips were sold. \$702.52 had been placed in the till.

During the period 1:30 - 4:30, business slackened off. 26 regular orders of fish were sold during this time. Four times as many regular orders had been sold during the lunch break. No jumbo portions were sold and the number of orders of chips declined to one fifth the number that had been sold during the lunch break. At 4:30 pm there was \$850.46 in Jerry's till.

What is the price of a regular sized order of fish at Jerry's? [5 marks]

**2.**  $ABCD$  is a square with side of length  $s$ . A circle, centre  $A$  and radius  $r$  is drawn so that the arc of this circle which lies within the square divides the square into two regions of equal area. Write  $r$  as a function of  $s$ . [6 marks]

**3.**(a) Solve the equation  $3^y = 10^y$ . [3 marks]

(b) Solve the equation  $3^y = 10$ . [3 marks]

(c) Write  $t \log_8 \sqrt{x} - 2 \log_8 y$  as a single logarithm. [4 marks]

**4.** Establish the identity  $2 \cot A = \cot \frac{A}{2} - \tan \frac{A}{2}$ . [5 marks]

**5.**  $ABC$  is a triangle, right angled at  $C$ . Let  $a, b, c$  denote the lengths of the sides opposite angles  $A, B, C$  respectively. Given that  $a = 1$ ,  $\angle B = 75^\circ$  and that  $\tan 75^\circ = 2 + \sqrt{3}$ , express  $b$  and  $c$  in the form  $p + q\sqrt{3}$  such that  $p = 2$  or  $\sqrt{2}$ . [8 marks]

**6.** Determine the function  $f(x)$  which satisfies all of the following conditions: [8 marks]

- (i)  $f(x)$  is a quadratic function.
- (ii)  $f(x + 2) = f(x) + x + 2$ .
- (iii)  $f(2) = 2$ .

**7.** Prove that if  $n$  is a positive integer (written in base 10) and that if 9 is a factor of  $n$ , then 9 is also a factor of the sum of the digits of  $n$ . [8 marks]

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Last number we gave the problems of the American Invitational Mathematics Examination. These problems and their solutions are copyrighted by the Committee on the American Mathematical Competitions of the Mathematical Association of America and may not be reproduced without permission. Full solutions, and additional copies of the problems, may be obtained for a nominal fee from Professor Walter E. Mientka, C.A.M.C. Executive Director, 917 Oldfather Hall, University of Nebraska, Lincoln, NE, U.S.A. 68588-0322.

### Solutions to the 1996 A.I.M.E.

1. 200	2. 340	3. 044	4. 166	5. 023
6. 049	7. 300	8. 799	9. 342	10. 159
11. 276	12. 058	13. 065	14. 768	15. 777

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That completes the Skoliad Corner for this issue. Please send me suitable contest materials, your students' nice solutions, and comments or advice for the future of the Corner.

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