THE ACADEMY CORNER

No. 3

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Spring is approaching in parts of Canada, and, as undergraduate students in North America prepare for final examinations, some find time to try their skills on some mathematics problems. Memorial University offers some modest prizes for the best students. How do you compare? Please send me your best solutions to these problems.

MEMORIAL UNIVERSITY OF NEWFOUNDLAND UNDERGRADUATE MATHEMATICS COMPETITION

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1. Prove that if n is a positive integer, then $\dfrac{n^2+3n+1}{n^2+4n+3}$ is an irreducible fraction.

2. A jar contains 7 blue balls, 9 red balls and 10 white balls. Balls are drawn at random one by one from the jar until either four balls of the same colour or at least two of each colour have been drawn. What is the largest number of balls that one may have to draw?

3. Find all functions $u(x)$ satisfying $u(x) = x + \displaystyle\int_0^{\frac{1}{2}} u(t)dt$.

4. Show that $(\sqrt{5}+2)^{\frac{1}{3}} - (\sqrt{5}-2)^{\frac{1}{3}}$ is a rational number and find its value.

5. In a quadrilateral $ABCD$ (vertices named in clockwise order), AC and BD intersect in X. You are given that $AB \parallel DC$, that AB is twice as long as CX and that AC is equal in length to DC. Show that AB and CD are equal in length (and hence $ABCD$ is a parallelogram).

6. Prove that among any thirteen distinct real numbers it is possible to choose two, x and y, such that $0 < \dfrac{x-y}{1+xy} < 2-\sqrt{3}$.

7. A coast guard boat is hunting a bootlegger in a fog. The fog rises disclosing the bootlegger 4 miles distant and immediately descends. The speed of the boat is 3 times that of the bootlegger, and it is known that the latter will immediately depart at full speed on a straight course of unknown direction. What course should the boat take in order to overtake the bootlegger?

Historical Titbit

Taken from a 1950's University Scholarship Paper.

A, C, B are collinear points.

Prove that there is one and only one point D such that $(ACBD)$ is harmonic.

The following result is stated in a book on geometry:

> if the pencils $O(ACBD)$ and $O'(ACBD)$ are harmonic and if A, C, B are collinear, then D lies on the line ACB.

Give an example to show that this result is not always true.

What alteration is required in order to make it true?

Prove the result after making this alteration.