PROBLEMS

Problem proposals and solutions should be sent to Bruce Shawyer, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. A1C 5S7. Proposals should be accompanied by a solution, together with references and other insights which are likely to be of help to the editor. When a submission is submitted without a solution, the proposer must include sufficient information on why a solution is likely. An asterisk ($\star$) after a number indicates that a problem was submitted without a solution.

In particular, original problems are solicited. However, other interesting problems may also be acceptable provided that they are not too well known, and references are given as to their provenance. Ordinarily, if the originator of a problem can be located, it should not be submitted without the originator's permission.

To facilitate their consideration, please send your proposals and solutions on signed and separate standard $8\frac{1}{2}''\times 11''$ or A4 sheets of paper. These may be typewritten or neatly hand-written, and should be mailed to the Editor-in-Chief, to arrive no later that **1 October 1996**. *They may also be sent by email to* **cruxeditor@cms.math.ca**. (*It would be appreciated if email proposals and solutions were written in LaTeX, preferably in LaTeX2e*). *Graphics files should be in epic format, or plain postscript. Solutions received after the above date will also be considered if there is sufficient time before the date of publication.*

2114. *Proposed by Toshio Seimiya, Kawasaki, Japan.*

$ABCD$ is a square with incircle Γ. A tangent ℓ to Γ meets the sides AB and AD and the diagonal AC at P, Q and R respectively. Prove that

$$\frac{AP}{PB}+\frac{AR}{RC}+\frac{AQ}{QD}=1.$$

2115. *Proposed by Toby Gee, student, The John of Gaunt School, Trowbridge, England.*

Find all polynomials f such that $f(p)$ is a prime for every prime p.

2116. *Proposed by Yang Kechang, Yueyang University, Hunan, China.*

A triangle has sides a, b, c and area F. Prove that

$$a^3b^4c^5 \geq \frac{25\sqrt{5}(2F)^6}{27}.$$

When does equality hold?

2117. *Proposed by Toshio Seimiya, Kawasaki, Japan.*

ABC is a triangle with $AB > AC$, and the bisector of $\angle A$ meets BC at D. Let P be an interior point of the side AC. Prove that $\angle BPD < \angle DPC$.

2118. *Proposed by Paul Yiu, Florida Atlantic University, Boca Raton, Florida, USA.*

The primitive Pythagorean triangle with sides 2547 and 40004 and hypotenuse 40085 has area 50945094, which is an 8-digit number of the form $abcdabcd$. Find another primitive Pythagorean triangle whose area is of this form.

2119. *Proposed by Hoe Teck Wee, student, Hwa Chong Junior College, Singapore.*

(a) Show that for any positive integer $m \geq 3$, there is a permutation of m 1's, m 2's and m 3's such that

  (i) no block of consecutive terms of the permutation (other than the entire permutation) contains equal numbers of 1's, 2's and 3's; and

  (ii) there is no block of m consecutive terms of the permutation which are all equal.

(b) For $m = 3$, how many such permutations are there?

2120. *Proposed by Marcin E. Kuczma, Warszawa, Poland.*

Let $A_1A_3A_5$ and $A_2A_4A_6$ be nondegenerate triangles in the plane. For $i = 1, \ldots, 6$ let ℓ_i be the perpendicular from A_i to line $A_{i-1}A_{i+1}$ (where of course $A_0 = A_6$ and $A_7 = A_1$). If ℓ_1, ℓ_3, ℓ_5 concur, prove that ℓ_2, ℓ_4, ℓ_6 also concur.

2121. *Proposed by Krzysztof Chelmiński, Technische Hochschule Darmstadt, Germany; and Waldemar Pompe, student, University of Warsaw, Poland.*

Let $k \geq 2$ be an integer. The sequence (x_n) is defined by $x_0 = x_1 = 1$ and

$$x_{n+1} = \frac{x_n^k + 1}{x_{n-1}} \qquad \text{for} \quad n \geq 1.$$

(a) Prove that for each positive integer $k \geq 2$ the sequence (x_n) is a sequence of integers.

(b) If $k = 2$, show that $x_{n+1} = 3x_n - x_{n-1}$ for $n \geq 1$.

(c)* Note that for $k = 2$, part (a) follows immediately from (b). Is there an analogous recurrence relation to the one in (b), not necessarily linear, which would give an immediate proof of (a) for $k \geq 3$?

2122. *Proposed by Shawn Godin, St. Joseph Scollard Hall, North Bay, Ontario.*

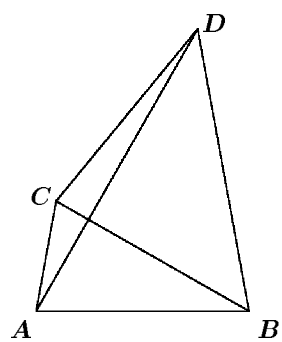
Little Sam is a unique child and his math marks show it. On four tests this year his scores out of 100 were all two-digit numbers made up of eight different non-zero digits. What's more, the average of these scores is the same as the average if each score is reversed (so 94 becomes 49, for example), and this average is an integer none of whose digits is equal to any of the digits in the scores. What is Sam's average?

2123. *Proposed by Sydney Bulman–Fleming and Edward T. H. Wang, Wilfrid Laurier University, Waterloo, Ontario.*

It is known (e.g., exercise 23, page 78 of Kenneth H. Rosen's *Elementary Number Theory and its Applications*, Third Edition) that every natural number greater than 6 is the sum of two relatively prime integers, each greater than 1. Find all natural numbers which can be expressed as the sum of *three* pairwise relatively prime integers, each greater than 1.

2124. *Proposed by Catherine Shevlin, Wallsend, England.*

Suppose that $ABCD$ is a quadrilateral with $\angle CDB = \angle CBD = 50^\circ$ and $\angle CAB = \angle ABD = \angle BCD$. Prove that $AD \perp BC$.



Mathematical Literacy

1. Who said: "To be able to read the great book of the universe, one must first understand its language, which is that of mathematics".

2. In referring to "the unreasonable effectiveness of mathematics in the natural sciences", who wrote: "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve".