The Academy Corner

No. 1

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With this issue, we start a new corner, which, for want of a better name, is being called "the Academy Corner". It will be concerned, in particular, with problem solving at the undergraduate level. This can take the form of a competition, a "problem of the week", a course in problem solving, etc. Your submissions and comments are welcome!

Many Universities hold their own undergraduate Mathematics Competitions. We invite subscribers to send us information about them for publication here. We also solicit nice solutions to these problems. Please send them to the Editor-in-Chief.

We shall begin with the competition with which the editor is most familiar: the Undergraduate Mathematics Competition at Memorial University of Newfoundland. This competition is used as primary information for choosing Memorial's team for the Atlantic Provinces Council on the Sciences Mathematics Competition (more on that in a later issue).

Memorial University Undergraduate Mathematics Competition 1995

Time allowed — 3 hours

1. Find all integer solutions of the equation $x^4 = y^2 + 71$.

2. (a) Show that $x^2 + y^2 \geq 2xy$ for all real numbers $x, y$.

   (b) Show that $a^2 + b^2 + c^2 \geq ab + bc + ca$ for all real numbers $a, b, c$.

3. Find the sum of the series

   $$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \ldots + \frac{99}{100!}.$$
4. If \( a, b, c, d \) are positive integers such that \( ad = bc \), prove that \( a^2 + b^2 + c^2 + d^2 \) is never a prime number.

5. Determine all functions \( f : \mathbb{R} \to \mathbb{R} \) which satisfy
\[
(x - y)f(x + y) - (x + y)f(x - y) = 4xy(x^2 - y^2)
\]
for all real numbers \( x, y \).

6. Assume that when a snooker ball strikes a cushion, the angle of incidence equals the angle of reflection.
   
   For any position of a ball \( A \), a point \( P \) on the cushion is determined as shown.
   
   Prove that if the ball \( A \) is shot at point \( P \), it will go into the pocket \( B \).

This concludes the first Academy Corner.

**Mathematical Literacy**

1. Who thought that the binary system would convince the Emperor of China to abandon Buddhism in favour of Christianity?

2. Who asked which king for one grain of wheat for the first square of a chess board, two grains for the second square, four grains for the third square, and so on?

3. In which well known painting, by whom, does a Magic Square appear?

4. Where was bread cut into "Cones, Cylinders, Parallelograms, and several other Mathematical Figures"?

5. Which mathematician said: "Philosophers count about two-hundred and eighty eight views of the sovereign good"?

Answers will be given in a subsequent issue.