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E U R E K A

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* * *

AN INTERESTING FAMILY OF PRIMITIVES

F.G.B. MASKELL, Algonquin College

The following problem appears on page 341 in *Calculus and Analytic Geometry*, by John A. Tierney (Allyn and Bacon, 1972):

Evaluate $\displaystyle\int \frac{(x^2-1)dx}{x^4+3x^2+1}$.

The answer given in the accompanying Solutions Manual is $\text{Tan}^{-1}(x+x^{-1}) + C$.

L. Sauvé pointed out to me that this solution is discontinuous at $x=0$, whereas the integrand is continuous for all real x; hence the proposed solution is not an acceptable answer to the problem. Investigation led to a number of apparently different correct solutions of which the simplest can be obtained as follows:

$$I_0 = \int \frac{(x^2-1)dx}{x^4+3x^2+1} = -\int \frac{(1-x^2)dx}{(x^2+1)^2+x^2} = -\int \frac{1}{1+\left(\frac{x}{x^2+1}\right)^2}\cdot d\left(\frac{x}{x^2+1}\right)$$

$$= -\phi(x) + C \qquad (1)$$

where

$$\phi(x) = \arctan\frac{x}{x^2+1}.$$

Substitution of $C=\alpha+C'$ in (1) yields the following family of inverse tangent functions:

$$I_\alpha = \alpha - \phi(x) + C'. \qquad (2)$$

An easy calculation shows that

$$\tan(\alpha-\phi(x)) = \frac{(x^2+1)\sin\alpha - x\cos\alpha}{(x^2+1)\cos\alpha + x\sin\alpha};$$

hence, provided α is restricted so that $|\alpha-\phi(x)| < \frac{\pi}{2}$, we have

$$I_\alpha = \arctan\frac{(x^2+1)\sin\alpha - x\cos\alpha}{(x^2+1)\cos\alpha + x\sin\alpha} + C'. \tag{3}$$

Since the discriminant of the denominator in (3) is $\sin^2\alpha - 4\cos^2\alpha$, continuity requires

$$|\alpha| < \arctan 2 = \frac{\pi}{2} - \arctan\frac{1}{2}. \tag{4}$$

If we note furthermore that

$$|\phi(x)| = \arctan\left|\frac{x}{x^2+1}\right| \le \arctan\frac{1}{2},$$

so that

$$-\frac{\pi}{2} < \alpha - \arctan\frac{1}{2} \le \alpha-\phi(x) \le \alpha+\arctan\frac{1}{2} < \frac{\pi}{2},$$

it becomes clear that (4) ensures the equivalence of (2) and (3). Since $C = \alpha + C'$, the two families described by (1) and (3) are in fact identical.

All expressions derived from (3) subject to restriction (4) describe the same family, despite such diversities in appearance as

$$-\arctan\frac{x}{x^2+1} + C', \qquad \text{for } \alpha = 0,$$

$$\arctan\frac{x^2-x+1}{x^2+x+1} + C', \qquad \text{for } \alpha = \frac{\pi}{4},$$

$$\arctan\frac{5x^2-12x+5}{12x^2+5x+12} + C', \qquad \text{for } \alpha = \arctan\frac{5}{12}.$$

If restriction (4) is not adhered to, the families resulting from (3) have discontinuities, and therefore do not constitute acceptable solutions to Tierney's problem; for example,

$$-\arctan\frac{2x^2+x+2}{(x-1)^2} + C', \qquad \text{for } \alpha = -\arctan 2,$$

$$\arctan\frac{2x^2-x+2}{(x+1)^2} + C', \qquad \text{for } \alpha = \arctan 2,$$

$$\arctan\left(x+\frac{1}{x}\right) + C', \qquad \text{for } \alpha = \frac{\pi}{2} \text{ (Tierney's solution).}$$

* * *

PROBLEMS--PROBLÈMES

Problem proposals, preferably accompanied by a solution, should be sent to the editor, whose address appears on page 9.

For the problems given below, solutions, if available, will appear in EUREKA No. 5, to be published around July 15, 1975. To facilitate their consideration, your solutions, typewritten or neatly handwritten on signed, separate sheets, should be mailed to the editor no later than July 1, 1975.

21. *Proposed by H.G.Dworschak, Algonquin College.*

What single standard mathematical symbol can be used with the digits 2 and 3 to make a number greater than 2 but less than 3?

22. *Proposed by H.G.Dworschak, Algonquin College.*

Numbers are written on little paper squares as shown in the figure.

1	2	7	9
3	4	5	8

Show how to make the sums of the two rows equal by moving just two of the pieces.

23. *Proposé par Léo Sauvé, Collège Algonquin.*

Déterminer s'il existe une suite $\{u_n\}$ d'entiers naturels telle que, pour $n = 1, 2, 3, \cdots$, on ait

$$2^{u_n} < 2n+1 < 2^{1+u_n}$$

24. *Proposed by Viktors Linis, University of Ottawa.*

A paper triangle has base 6 cm and height 2 cm. Show that by three or fewer cuts the sides can cover a cube of edge 1 cm.

25. *Proposed by Viktors Linis, University of Ottawa.*

Find the smallest positive value of $36^k - 5^\ell$ where k and ℓ are positive integers.

26. *Proposed by Viktors Linis, University of Ottawa.*

Given n integers. Show that one can select a subset of these numbers and insert plus or minus signs so that the number obtained is divisible by n.

27. *Proposé par Léo Sauvé, Collège Algonquin.*

Soient A, B, et C les angles d'un triangle. Il est facile de vérifier que si $A = B = 45^\circ$, alors

$$\cos A \cos B + \sin A \sin B \sin C = 1$$

La proposition réciproque est-elle vraie?

28. *Proposed by Léo Sauvé, Algonquin College.*

If 7% of the population escapes getting a cold during any given year, how many days must the average inhabitant expect to wait from one cold to the next?

29. *Proposed by Viktors Linis, University of Ottawa.*

Cut a square into a minimal number of triangles with all angles acute.

30. *Proposed by Léo Sauvé, Algonquin College.*

Let a, b, and c denote three distinct integers and let P denote a polynomial having all integral coefficients. Show that it is impossible that $P(a) = b$, $P(b) = c$, and $P(c) = a$. (*Third USA Mathematical Olympiad - May 7, 1974*)

* * *

SOLUTIONS

1. *Proposed by Léo Sauvé, Algonquin College.*

75 cows have in 12 days grazed all the grass in a 60-acre pasture, and 81 cows have in 15 days grazed all the grass in a 72-acre pasture. How many cows can in 18 days graze all the grass in a 96-acre pasture? (*Newton*)

Solution by the proposer.

The proposed problem is a special case of the following problem, which appeared in Newton's *Arithmetica Universalis* (1707):

a cows graze b acres bare in c days,

a' cows graze b' acres bare in c' days,

a'' cows graze b'' acres bare in c'' days;

what relation exists between the nine magnitudes a to c''?

This problem, together with a solution essentially the same as the one given below, can be found on page 9 of *100 Great Problems of Elementary Mathematics*, by Heinrich Dörrie (Dover, 1965).

The problem can be solved only if certain reasonable assumptions are made. We assume constant the daily ration of each cow, the number r of rations per acre initially present in each pasture, and the number s of rations per acre growing daily in each pasture.

In the first instance, br rations are initially present in the pasture, bcs rations grow during the period of grazing, and the cows consume ca rations; hence we must have

$$br + bcs - ca = 0,$$

and similarly

$$b'r + b'c's - c'a' = 0,$$
$$b''r + b''c''s - c''a'' = 0.$$

Now the homogeneous linear system

$$\begin{cases} bx + bcy + caz = 0 \\ b'x + b'c'y + c'a'z = 0 \\ b''x + b''c''y + c''a''z = 0 \end{cases}$$

has the nonzero solution $(r,s,-1)$; hence the determinant of the system must vanish, that is,

$$\begin{vmatrix} b & bc & ca \\ b' & b'c' & c'a' \\ b'' & b''c'' & c''a'' \end{vmatrix} = 0, \qquad (1)$$

and this is the required relation. If any eight of the nine magnitudes a to c'' are known, the ninth can be determined. In the proposed problem, we have

$$a = 75,\ a' = 81,\ b = 60,\ b' = 72,\ b'' = 96,\ c = 12,\ c' = 15,\ c'' = 18,$$

and relation (1) then yields $a'' = 100$.

Also solved by Richard Atlani, Algonquin College; H.G. Dworschak, Algonquin College; G.D. Kaye, Department of National Defence; André Ladouceur, École Secondaire De La Salle; and F.G.B. Maskell, Algonquin College.

2. *Proposed by Léo Sauvé, Algonquin College.*

A rectangular array of m rows and n columns contains mn distinct real numbers. For $i = 1, 2, \cdots m$, let s_i denote the smallest number of the i^{th} row;

and for $j = 1, 2, \cdots n$, let l_j denote the largest number of the j^{th} column. Let $A = \max\{s_i\}$ and $B = \min\{l_j\}$. Compare A and B.

Solution by the proposer.

Since the mn numbers are all distinct, there is a uniquely determined number C which lies at the intersection of the row of A and the column of B. (C may perhaps coincide with A or B). We must have $A \le C$ since A is the smallest number in its own row, and $C \le B$ since B is the largest number in its own column. Thus $A \le B$.

Also solved by H.G. Dworschak, Algonquin College; G.D. Kaye, Department of National Defence; André Ladouceur, École Secondaire De La Salle; and F.G.B. Maskell, Algonquin College.

3. *Proposed by H.G. Dworschak, Algonquin College.*

Prove that in any set of ten different two-digit numbers one can select two disjoint subsets such that the sum of numbers in each of the subsets is the same.

Solution d'André Ladouceur, École Secondaire De La Salle.

Tout ensemble de dix nombres distincts à deux chiffres admet $2^{10} - 1 = 1023$ sous-ensembles distincts non vides, mais le nombre de différentes sommes possibles est au plus 936, puisque la plus petite somme possible est 10 et la plus grande est $90 + 91 + \cdots + 99 = 945$. Il existe donc des sous-ensembles distincts qui donnent la même somme. Si deux tels sous-ensembles ne sont pas disjoints, on obtient des sous-ensembles disjoints de même somme en éliminant de chacun d'eux les éléments communs.

Comment by Viktors Linis, University of Ottawa.

10 is the smallest number of elements for which this problem would have a solution; for example, with 9 we would have only $2^9 - 1 = 511$ different nonempty subsets, but the number of possible sums would be 846. For two-digit number sets the necessary and sufficient condition is the inequality

$$2^n - 1 > \frac{199n - n^2 - 18}{2}$$

which holds only for $n > 9$.

Also solved by Viktors Linis, University of Ottawa; Léo Sauvé, Collège Algonquin; and the proposer.

4. *Proposed by Léo Sauvé, Algonquin College.*

It is easy to verify that $2\sqrt{3}+i$ is a cube root of $18\sqrt{3}+35i$. What are the other two cube roots?

Solution by H.G. Dworschak, Algonquin College.

Using the two imaginary cube roots of unity,

$$\omega = \frac{1}{2}(-1+i\sqrt{3}), \qquad \omega^2 = \frac{1}{2}(-1-i\sqrt{3}),$$

we find the two required roots to be

$$(2\sqrt{3}+i)\omega = \frac{1}{2}(-3\sqrt{3}+5i),$$

$$(2\sqrt{3}+i)\omega^2 = \frac{1}{2}(-\sqrt{3}-7i).$$

Also solved by G.D. Kaye, Department of National Defence; F.G.B. Maskell, Algonquin College; and the proposer.

5. *Proposed by F.G.B. Maskell, Algonquin College.*

Prove that, if (a,b,c) and (a',b',c') are primitive Pythagorean triples, with $a>b>c$ and $a'>b'>c'$, then either

$$aa' \pm (bc' - cb') \quad \text{or} \quad aa' \pm (bb' - cc')$$

are perfect squares.

Solution by the proposer, H.G. Dworschak, and Léo Sauvé, all from Algonquin College (Editor's composite).

The primitive Pythagorean triples (x,y,z) are characterized by

$$\text{g.c.d.}\{x,y,z\} = 1 \quad \text{and} \quad x^2+y^2 = z^2.$$

It is known from number theory that they all satisfy

$$\{x,y\} = \{2rs, r^2-s^2\}$$

for some positive integers r and s, and that then $z=r^2+s^2$.

For the given Pythagorean triples (a,b,c) and (a',b',c'), where $a>b>c$ and $a'>b'>c'$, we must have, for some positive integers r, s, u, v,

$$(1)\ \begin{cases} a = r^2+s^2 \\ b = 2rs \\ c = r^2-s^2 \end{cases} \qquad \text{or} \qquad (2)\ \begin{cases} a = r^2+s^2 \\ b = r^2-s^2 \\ c = 2rs \end{cases}$$

and

$$(3)\quad \begin{cases} a' = u^2 + v^2 \\ b' = 2uv \\ c' = u^2 - v^2 \end{cases} \qquad \text{or} \qquad (4)\quad \begin{cases} a' = u^2 + v^2 \\ b' = u^2 - v^2 \\ c' = 2uv \end{cases}$$

Depending upon whether (1) and (3), (2) and (4), (1) and (4), or (2) and (3) hold, each of the proposed quantities, and of a few others we are finding at the same time, is either a square (S) or twice a square ($2S$). This can easily be verified by substituting the values from (1), (2), (3), or (4) into the chosen quantity. The results are set out in the table below, where S stands for one of the quantities

$$[r(u \pm v) \pm s(u \pm v)]^2, \quad (ru \pm sv)^2, \quad (rv \pm su)^2.$$

	(1) and (3)	(2) and (4)	(1) and (4)	(2) and (3)
$aa' + bc' + cb'$	S	S	$2S$	$2S$
$aa' + bc' - cb'$	S	S	$2S$	$2S$
$aa' - bc' + cb'$	S	S	$2S$	$2S$
$aa' - bc' - cb'$	S	S	$2S$	$2S$
$aa' + bb' + cc'$	$2S$	$2S$	S	S
$aa' + bb' - cc'$	$2S$	$2S$	S	S
$aa' - bb' + cc'$	$2S$	$2S$	S	S
$aa' - bb' - cc'$	$2S$	$2S$	S	S

6. *Proposed by Léo Sauvé, Algonquin College.*

(a) If n is a given nonnegative integer, how many distinct nonnegative integer solutions are there for each of the following equations?

$$x + y = n, \qquad x + y + z = n, \qquad x + y + z + t = n.$$

(b) Use (a) to conjecture and then prove a formula for the number of distinct nonnegative integer solutions of the equation

$$x_1 + x_2 + \ldots + x_r = n.$$

Solution by the proposer.

(a) The first equation has, for $x = 0, 1, 2, \ldots, n$, the $n + 1$ solutions in which the value of y is, respectively, $n, n - 1, n - 2, \ldots, 0$. The number of distinct solutions (x,y) is thus

$$a_n = n + 1,$$

and this is valid also for $n = 0$.

The second equation has, for $x = p$, the a_{n-p} solutions of $y + z = n - p$. The number of distinct solutions (x, y, z) is thus

$$\begin{aligned} b_n &= a_0 + a_1 + \ldots + a_n \\ &= 1 + 2 + \ldots + (n + 1) \\ &= \frac{(n + 1)(n + 2)}{2}, \end{aligned}$$

and this is valid also for $n = 0$.

The last equation has, for $x = p$, the b_{n-p} solutions of $y + z + t = n - p$. The number of distinct solutions (x, y, z, t) is thus

$$\begin{aligned} c_n &= b_0 + b_1 + \ldots + b_n \\ &= \frac{1}{2}\sum_{k=1}^{n} k^2 + \frac{3}{2}\sum_{k=1}^{n} k + (n + 1) \\ &= \frac{(n + 1)(n + 2)(n + 3)}{6} \end{aligned}$$

and this is valid also for $n = 0$

(b) The conjecture suggested by (a) is, of course, that the number of distinct solutions $(x_1, x_2, \ldots, x_r)$ is

$$\frac{(n + 1)(n + 2)\ldots(n + r - 1)}{(r - 1)!} = \binom{n + r - 1}{r - 1}$$

To see that this formula is correct we observe that the number of distinct solutions is the same as the number of terms in y^n in the product

$$(1+a_1y+a_1^2y^2+\ldots)(1+a_2y+a_2^2y^2+\ldots)\ldots(1+a_ry+a_r^2y^2+\ldots).$$

For the term

$$(a_1^{\alpha_1} a_2^{\alpha_2} \ldots a_r^{\alpha_r}) y^n$$

occurs in the product if and only if

$$\alpha_1 + \alpha_2 + \ldots + \alpha_r = n,$$

that is, if and only if $(\alpha_1, \alpha_2, \ldots \alpha_r)$ is a solution of our equation. Since we want to find only the *number* of solutions and not the solutions themselves, we can set $a_1 = a_2 = \ldots = a_r = 1$ and find the coefficient of y^n in the product

$$(1 + y + y^2 + \ldots)^r = (1 - y)^{-r}.$$

This coefficient turns out to be, as expected,

$$\binom{n+r-1}{r-1}$$

Also solved by H.G. Dworschak, Algonquin College.

7. *Proposed by H.G. Dworschak, Algonquin College.*

Find a fifth degree polynomial $P(x)$ such that $P(x) + 1$ is divisible by $(x-1)^3$ and $P(x) - 1$ is divisible by $(x+1)^3$.

Solution by Viktors Linis, University of Ottawa.

The fourth degree polynomial $P'(x)$ is divisible both by $(x-1)^2$ and by $(x+1)^2$; hence it must be of the form

$$P'(x) = A(x^2 - 1)^2 = A(x^4 - 2x^2 + 1),$$

with constant $A \neq 0$, and so

$$P(x) = A\left(\frac{x^5}{5} - \frac{2x^3}{3} + x\right) + C.$$

The conditions $P(1) = -1$ and $P(-1) = 1$ give two linear equations in A and C. Solving these we find $A = -\frac{15}{8}$ and $C = 0$. The required polynomial is

$$P(x) = -\frac{3}{8}x^5 + \frac{5}{4}x^3 - \frac{15}{8}x.$$

Also solved by G. D. Kaye, Department of National Defence; André Ladouceur, École Secondaire De La Salle; F.G.B.Maskell, Algonquin College; Léo Sauvé, Collège Algonquin; and the proposer.

8. *Proposé par Jacques Marion, Université d'Ottawa.*

Etudier la convergence de la suite $\{a_n\}$ définie par

$$a_n = \sqrt{1+\sqrt{1+\sqrt{1+\ldots+\sqrt{1}}}}, \qquad (n \text{ radicaux})$$

et déterminer $\lim\limits_{n\to\infty} a_n$ si elle existe.

Solution d'André Ladouceur, École Secondaire De La Salle.

La suite $\{a_n\}$ est croissante, car $a_2 = \sqrt{2} > a_1$ et, pour $n \geq 2$,

$$a_{n+1} - a_n = \sqrt{1+a_n} - a_n = \frac{1+a_n-a_n^2}{\sqrt{1+a_n}+a_n} = \frac{a_n - a_{n-1}}{\sqrt{1+a_n}+a_n},$$

de sorte que $a_n > a_{n-1} \Longrightarrow a_{n+1} > a_n$. La suite est aussi bornée, car $a_1 = 1 < 2$ et

$$a_n < 2 \Longrightarrow a_{n+1} = \sqrt{1+a_n} < \sqrt{1+2} < 2.$$

La suite admet donc une limite λ, qui doit vérifier

$$\lambda = \lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{1+a_{n-1}} = \sqrt{1+\lim_{n\to\infty} a_{n-1}} = \sqrt{1+\lambda};$$

λ est donc la racine positive de l'equation $x^2 - x - 1 = 0$, c'est-à- dire

$$\lim_{n\to\infty} a_n = \lambda = \frac{1+\sqrt{5}}{2} \doteq 1.618. .$$

On reconnaît en λ le *nombre d'or* des Grecs.

Also solved by G.D. Kaye, Department of National Defence; Léo Sauvé, Collège Algonquin; John Thomas, Digital Methods Ltd; and the proposer.

9. *Proposé par Jacques Marion, Université d'Ottawa.*

Étudier la convergence de la suite $\{b_n\}$ définie par

$$b_n = \sqrt{1+\sqrt{2+\sqrt{3+\ldots+\sqrt{n}}}}.$$

Solution du proposeur.

La suite $\{b_n\}$ est strictement croissante, car si l'on rationnalise n fois le numérateur de la quantité $b_{n+1} - b_n$, on obtient

$$b_{n+1} - b_n = \frac{\sqrt{n+1}}{P} > 0,$$

car le dénominateur P est une quantité positive.

Nous allons maintenant montrer que la suite $\{b_n\}$ est bornée, et nous serons ensuite en mesure d'affirmer qu'elle est convergente.

On a pour $n \geq 2$,

$$\frac{b_n^2 - 1}{\sqrt{2}} = \sqrt{1 + \frac{1}{2}\sqrt{3 + \sqrt{4 + \ldots + \sqrt{n}}}}$$

$$= \sqrt{1 + \sqrt{\frac{3}{2^2} + \frac{1}{2^2}\sqrt{4 + \ldots + \sqrt{n}}}}$$

$$= \ldots$$

$$= \sqrt{1 + \sqrt{\frac{3}{2^2} + \sqrt{\frac{4}{2^4} + \sqrt{\frac{5}{2^8} + \ldots + \sqrt{\frac{n}{2^{2^{n-2}}}}}}}}$$

$$\leq \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots + \sqrt{1}}}} \qquad (n\text{-1 radicaux})$$

$$< \lambda = \frac{1 + \sqrt{5}}{2},$$

ou λ est le nombre d'or obtenu au problème précédent (No. 8), puisque $\frac{n}{2^{2^{n-2}}} \leq 1$ pour $n \geq 2$. Il résulte, pour $n \geq 2$,

$$b_n < \sqrt{1 + \sqrt{2}\lambda} \doteq 1.813$$

La suite $\{b_n\}$ est donc croissante, bornée, et par conséquent, convergente.

Also solved by F.G.B. Maskell, Algonquin College.

10. *Proposé par Jacques Marion, Université d'Ottawa.*

On sait que l'équation $e^x = x$ n'a pas de racine réelle; mais l'équation $e^z = z$ a-t-elle des racines *complexes?*

Solution de Léo Sauvé, Collège Algonquin.

Pour $z = x + iy$, l'equation

$$e^z = z \qquad (1)$$

devient

$$e^x(\cos y + i\sin y) = x + iy;$$

elle équivaut donc au système

$$\begin{cases} e^x \cos y = x, \\ e^x \sin y = y. \end{cases} \qquad (2)$$

Il résulte immédiatement de (2) que

i) $\sin y \neq 0$ et $y \neq 0$, car

$$\sin y = 0 \Longleftrightarrow y = 0 \Longrightarrow e^x = x,$$

et l'on sait que cette dernière équation n'a pas de racine réelle;

ii) $\cos y \neq 0$ et $x \neq 0$, car

$$\cos y = 0 \Longleftrightarrow x = 0 \Longrightarrow \sin y = y \Longrightarrow y = 0,$$

et l'on peut dès lors conclure que l'équation (1) n'admet ni racine réelle, ni racine imaginaire pure;

iii) si le couple (x,y) vérifie (2), il en est de même du couple $(x,-y)$. On peut donc se limiter à rechercher les solutions (x,y) de (2) telles que $y>0$, c'est-à-dire les solutions du système

$$\begin{cases} e^x \cos y = x, \\ e^x \sin y = y, \\ \qquad y > 0. \end{cases} \qquad (3)$$

Notons α l'unique racine de l'équation $e^x + x = 0$. On trouve facilement que le système (3) est équivalent au système

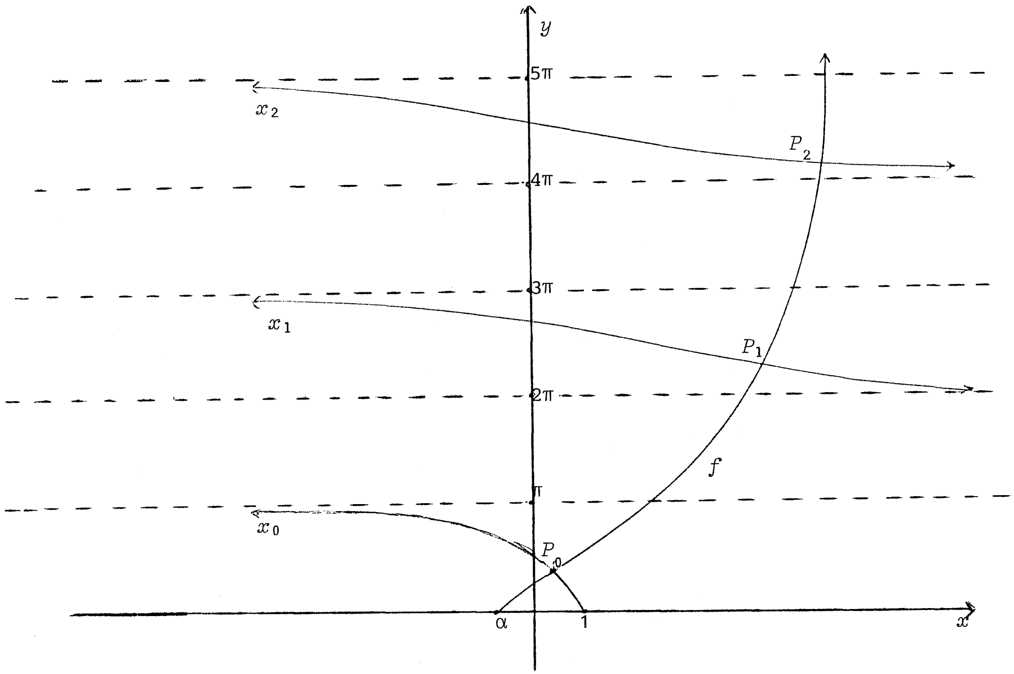
$$\begin{cases} y = \sqrt{e^{2x} - x^2} = f(x), & x > \alpha \doteq -0.56714, \\ x = y \cot y = g(y), & 2k\pi < y < (2k+1)\pi, \quad k = 0, 1, 2, \ldots, \\ \qquad \sin y > 0 . \end{cases}$$

On détermine par calcul élémentaire que la fonction f est croissante sur tout son domaine et tend vers l'infini avec x. On trouve de même que chaque branche $x_k = g(y)$ de la fonction g est décroissante sur tout son domaine $2k\pi < y < (2k+1)\pi$. De plus, sauf pour x_0, chaque x_k a pour image l'ensemble de tous les réels. Quant à x_0, puisque $\lim_{y \to 0^+} x_0 = 1$, son image est l'ensemble des réels inférieurs à 1.

On conclut que le graphe de f rencontre chaque branche x_k de g en un point unique P_k. Les coordonnées de ces points $P_0, P_1, P_2, \ldots$, ainsi que celles des points symétriques par rapport à l'axe des x, sont les solutions du système (2).

On doit employer des méthodes numériques pour trouver des valeurs approchées des coordonnées de ces points. Par exemple, on trouve $P_0 \doteq (0.31813, 1.33723)$.

La représentation graphique approximative qui suit illustre bien la situation.



Also solved by G.D. Kaye, Department of National Defence; F.G.B. Maskell, Algonquin College; and the proposer.

Editor's comment.

The proposer used Hadamard's Factorization Theorem to prove, more generally, that the equation $e^{\lambda z} = p(z)$, where $\lambda > 0$ and $p(z)$ is a polynomial with complex coefficients, has infinitely many complex roots. But the proof is one of existence only; it gives no clue as to the location of the roots.

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The editor is grateful to Dr. Viktors Linis, University of Ottawa, for a list of apt quotations which will, without further acknowledgment, grace the pages of future issues of EUREKA.

The next EUREKA Bull Session (no dinner this time) will be held at 7 p.m. on Tuesday, June 10, 1975, in room C-426 of the Colonel By Campus of Algonquin College, 281 Echo Drive, Ottawa.