The shifted classical circulant and skew circulant splitting iterative methods for Toeplitz matrices
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Abstract. It is known that every Toeplitz matrix $T$ enjoys a circulant and skew circulant splitting (denoted by CSCS) i.e., $T = C - S$ with $C$ a circulant matrix and $S$ a skew circulant matrix. Based on the variant of such a splitting (also referred to as CSCS), we first develop classical CSCS iterative methods and then introduce shifted CSCS iterative methods for solving hermitian positive definite Toeplitz systems in this paper. The convergence of each method is analyzed. Numerical experiments show that the classical CSCS iterative methods work slightly better than the Gauss-Seidel (GS) iterative methods if the CSCS is convergent, and that there is always a constant $\alpha$ such that the shifted CSCS iteration converges much faster than the Gauss-Seidel iteration, no matter whether the CSCS itself is convergent or not.