Characterizations of Besov-Type and Triebel-Lizorkin-Type Spaces via Averages on Balls
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Abstract. Let \( \ell \in \mathbb{N} \) and \( \alpha \in (0, 2\ell) \). In this article, the authors establish equivalent characterizations of Besov-type spaces, Triebel-Lizorkin-type spaces and Besov-Morrey spaces via the sequence \( \{ f - B_{\ell,2^{-k}} f \}_k \) consisting of the difference between \( f \) and the ball average \( B_{\ell,2^{-k}} f \). These results give a way to introduce Besov-type spaces, Triebel-Lizorkin-type spaces and Besov-Morrey spaces with any smoothness order on metric measure spaces. As special cases, the authors obtain a new characterization of Morrey-Sobolev spaces and \( Q_\alpha \) spaces with \( \alpha \in (0,1) \), which are of independent interest.