The thickness of the Cartesian product of two graphs
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Abstract. The thickness of a graph $G$ is the minimum number of planar subgraphs whose union is $G$. A $t$-minimal graph is a graph of thickness $t$ which contains no proper subgraph of thickness $t$. In this paper, upper and lower bounds are obtained for the thickness, $t(G \square H)$, of the Cartesian product of two graphs $G$ and $H$, in terms of the thickness $t(G)$ and $t(H)$. Furthermore, the thickness of the Cartesian product of two planar graphs and of a $t$-minimal graph and a planar graph are determined. By using a new planar decomposition of the complete bipartite graph $K_{4k,4k}$, the thickness of the Cartesian product of two complete bipartite graphs $K_{n,n}$ and $K_{n,n}$ is also given, for $n \neq 4k + 1$. 