Homological Properties Relative to Injectively Resolving Subcategories
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Abstract. Let $\mathcal{E}$ be an injectively resolving subcategory of left $R$-modules. A left $R$-module $M$ (resp. right $R$-module $N$) is called $\mathcal{E}$-injective (resp. $\mathcal{E}$-flat) if $\text{Ext}_R^1(G,M) = 0$ (resp. $\text{Tor}_R^1(N,G) = 0$) for any $G \in \mathcal{E}$. Let $\mathcal{E}$ be a covering subcategory. We prove that a left $R$-module $M$ is $\mathcal{E}$-injective if and only if $M$ is a direct sum of an injective left $R$-module and a reduced $\mathcal{E}$-injective left $R$-module. Suppose $\mathcal{F}$ is a preenveloping subcategory of right $R$-modules such that $\mathcal{E}^+ \subseteq \mathcal{F}$ and $\mathcal{F}^+ \subseteq \mathcal{E}$. It is shown that a finitely presented right $R$-module $M$ is $\mathcal{E}$-flat if and only if $M$ is a cokernel of an $\mathcal{F}$-preenvelope of a right $R$-module. In addition, we introduce and investigate the $\mathcal{E}$-injective and $\mathcal{E}$-flat dimensions of modules and rings. We also introduce $\mathcal{E}$-(semi)hereditary rings and $\mathcal{E}$-von Neumann regular rings and characterize them in terms of $\mathcal{E}$-injective and $\mathcal{E}$-flat modules.