Factoring a quadratic operator as a product of two positive contractions
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Abstract. Let $T$ be a quadratic operator on a complex Hilbert space $H$. We show that $T$ can be written as a product of two positive contractions if and only if $T$ is of the form

$$aI \oplus bI \oplus \begin{pmatrix} aI & P \\ 0 & bI \end{pmatrix} \quad \text{on} \quad H_1 \oplus H_2 \oplus (H_3 \oplus H_3)$$

for some $a, b \in [0, 1]$ and strictly positive operator $P$ with $\|P\| \leq |\sqrt{a} - \sqrt{b}|\sqrt{(1 - a)(1 - b)}$.

Also, we give a necessary condition for a bounded linear operator $T$ with operator matrix $\begin{pmatrix} T_1 & T_3 \\ 0 & T_2 \end{pmatrix}$ on $H \oplus K$ that can be written as a product of two positive contractions.