Constructing Double Magma on Groups Using Commutation Operations  
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Abstract. A magma \((M, \star)\) is a nonempty set with a binary operation. A double magma \((M, \star, \bullet)\) is a nonempty set with two binary operations satisfying the interchange law, \((w \star x) \bullet (y \star z) = (w \bullet y) \star (x \bullet z)\). We call a double magma proper if the two operations are distinct and commutative if the operations are commutative. A double semigroup, first introduced by Kock, is a double magma for which both operations are associative. Given a non-trivial group \(G\) we define a system of two magma \((G, \star, \bullet)\) using the commutator operations \(x \star y = [x, y] = x^{-1}y^{-1}xy\) and \(x \bullet y = [y, x]\). We show that \((G, \star, \bullet)\) is a double magma if and only if \(G\) satisfies the commutator laws \([x, y; x, z] = 1\) and \([w, x; y, z]^2 = 1\). We note that the first law defines the class of 3-metabelian groups. If both these laws hold in \(G\), the double magma is proper if and only if there exist \(x_0, y_0 \in G\) for which \([x_0, y_0]^2 \neq 1\). This double magma is a double semigroup if and only if \(G\) is nilpotent of class two. We construct a specific example of a proper double semigroup based on the dihedral group of order 16. In addition we comment on a similar construction for rings using Lie commutators.