

Countable dense homogeneity in powers of zero-dimensional definable spaces

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Abstract. We show that, for a coanalytic subspace X of 2^ω , the countable dense homogeneity of X^ω is equivalent to X being Polish. This strengthens a result of Hrušák and Zamora Avilés. Then, inspired by results of Hernández-Gutiérrez, Hrušák and van Mill, using a technique of Medvedev, we construct a non-Polish subspace X of 2^ω such that X^ω is countable dense homogeneous. This gives the first ZFC answer to a question of Hrušák and Zamora Avilés. Furthermore, since our example is consistently analytic, the equivalence result mentioned above is sharp. Our results also answer a question of Medini and Milovich. Finally, we show that if every countable subset of a zero-dimensional separable metrizable space X is included in a Polish subspace of X then X^ω is countable dense homogeneous.