Abstract. An inverse semigroup $S$ is combinatorially factorizable if $S = TG$ where $T$ is a combinatorial (i.e., $\mathcal{H}$ is the equality relation) inverse subsemigroup of $S$ and $G$ is a subgroup of $S$. This concept was introduced and studied by Mills, especially in the case when $S$ is cryptic (i.e., $\mathcal{H}$ is a congruence on $S$). Her approach is mainly analytical considering subsemigroups of a cryptic inverse semigroup.

We start with a combinatorial inverse monoid and a factorizable Clifford monoid and from an action of the former on the latter construct the semigroups in the title. As a special case, we consider semigroups which are direct products of a combinatorial inverse monoid and a group.