Asymptotic Improvements of Lower Bounds for the Least Common Multiples of Arithmetic Progressions

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Abstract. For relatively prime positive integers $u_0$ and $r$, we consider the least common multiple $L_n := \text{lcm}(u_0, u_1, \ldots, u_n)$ of the finite arithmetic progression $\{u_k := u_0 + kr\}_{k=0}^n$. We derive new lower bounds on $L_n$ that improve upon those obtained previously when either $u_0$ or $n$ is large. When $r$ is prime, our best bound is sharp up to a factor of $n + 1$ for $u_0$ properly chosen, and is also nearly sharp as $n \to \infty$. 