On the theorem of the primitive element with applications to the representation theory of associative and Lie algebras

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Abstract. We describe all finite dimensional uniserial representations of a commutative associative (resp. abelian Lie) algebra over a perfect (resp. sufficiently large perfect) field. In the Lie case the size of the field depends on the answer to following question, considered and solved in this paper. Let $K/F$ be a finite separable field extension and let $x, y \in K$. When is $F[x, y] = F[\alpha x + \beta y]$ for some non-zero elements $\alpha, \beta \in F$?