A problem on edge-magic labelings of cycles
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Abstract. Kotzig and Rosa defined in 1970 the concept of edge-magic labelings as follows: let $G$ be a simple $(p, q)$-graph (that is, a graph of order $p$ and size $q$ without loops or multiple edges). A bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}$ is an edge-magic labeling of $G$ if $f(u) + f(uv) + f(v) = k$, for all $uv \in E(G)$. A graph that admits an edge-magic labeling is called an edge-magic graph, and $k$ is called the magic sum of the labeling. An old conjecture of Godbold and Slater sets that all possible theoretical magic sums are attained for each cycle of order $n \geq 7$. Motivated by this conjecture, we prove that for all $n_0 \in \mathbb{N}$, there exists $n \in \mathbb{N}$, such that the cycle $C_n$ admits at least $n_0$ edge-magic labelings with at least $n_0$ mutually distinct magic sums. We do this by providing a lower bound for the number of magic sums of the cycle $C_n$, depending on the sum of the exponents of the odd primes appearing in the prime factorization of $n$. 