On 3-manifolds with Torus- or Klein-bottle Category Two
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Abstract. A subset $W$ of a closed manifold $M$ is $K$-contractible, where $K$ is a torus or Kleinbottle, if the inclusion $W \to M$ factors homotopically through a map to $K$. The image of $\pi_1(W)$ (for any base point) is a subgroup of $\pi_1(M)$ that is isomorphic to a subgroup of a quotient group of $\pi_1(K)$. Subsets of $M$ with this latter property are called $\mathcal{G}_K$-contractible. We obtain a list of the closed 3-manifolds that can be covered by two open $\mathcal{G}_K$-contractible subsets. This is applied to obtain a list of the possible closed prime 3-manifolds that can be covered by two open $K$-contractible subsets.