Duality of Preenvelopes and Pure Injective Modules
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Abstract. Let $R$ be an arbitrary ring and $(-)^+ = \text{Hom}_\mathbb{Z}(-, \mathbb{Q}/\mathbb{Z})$ where $\mathbb{Z}$ is the ring of integers and $\mathbb{Q}$ is the ring of rational numbers, and let $\mathcal{C}$ be a subcategory of left $R$-modules and $\mathcal{D}$ a subcategory of right $R$-modules such that $X^+ \in \mathcal{D}$ for any $X \in \mathcal{C}$ and all modules in $\mathcal{C}$ are pure injective. Then a homomorphism $f : A \to C$ of left $R$-modules with $C \in \mathcal{C}$ is a $\mathcal{C}$-$(pre)$envelope of $A$ provided $f^+ : C^+ \to A^+$ is a $\mathcal{D}$-$(pre)$cover of $A^+$. Some applications of this result are given.