The \( f \)-Chromatic Index of a Graph Whose \( f \)-Core has Maximum Degree 2

S. Akbari, M. Chavooshi, M. Ghanbari, and S. Zare

Abstract. Let \( G \) be a graph. The minimum number of colors needed to color the edges of \( G \) is called the chromatic index of \( G \) and is denoted by \( \chi'(G) \). It is well-known that \( \Delta(G) \leq \chi'(G) \leq \Delta(G) + 1 \), for any graph \( G \), where \( \Delta(G) \) denotes the maximum degree of \( G \). A graph \( G \) is said to be Class 1 if \( \chi'(G) = \Delta(G) \) and Class 2 if \( \chi'(G) = \Delta(G) + 1 \). Also, \( G_\Delta \) is the induced subgraph on all vertices of degree \( \Delta(G) \). Let \( f : V(G) \rightarrow \mathbb{N} \) be a function. An \( f \)-coloring of a graph \( G \) is a coloring of the edges of \( E(G) \) such that each color appears at each vertex \( v \in V(G) \) at most \( f(v) \) times. The minimum number of colors needed to \( f \)-color \( G \) is called the \( f \)-chromatic index of \( G \) and is denoted by \( \chi_f'(G) \). It was shown that for every graph \( G \), \( \Delta_f(G) \leq \chi_f'(G) \leq \Delta_f(G) + 1 \), where \( \Delta_f(G) = \max_{v \in V(G)} \left\lceil \frac{d_G(v)}{f(v)} \right\rceil \). A graph \( G \) is said to be \( f \)-Class 1 if \( \chi_f'(G) = \Delta_f(G) \), and \( f \)-Class 2, otherwise. Also, \( G_{\Delta_f} \) is the induced subgraph of \( G \) on \( \{v \in V(G) : \frac{d_G(v)}{f(v)} = \Delta_f(G)\} \). Hilton and Zhao showed that if \( G_\Delta \) has maximum degree two and \( G \) is Class 2, then \( G \) is critical, \( G_\Delta \) is a disjoint union of cycles and \( \delta(G) = \Delta(G) - 1 \), where \( \delta(G) \) denotes the minimum degree of \( G \), respectively. In this paper, we generalize this theorem to \( f \)-coloring of graphs. Also, we determine the \( f \)-chromatic index of a connected graph \( G \) with \( |G_{\Delta_f}| \leq 4 \).