Quasiconvexity and Density Topology
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Abstract. We prove that if $f : \mathbb{R}^N \to \overline{\mathbb{R}}$ is quasiconvex and $U \subset \mathbb{R}^N$ is open in the density topology, then $\sup_U f = \text{ess sup}_U f$, while $\inf_U f = \text{ess inf}_U f$ if and only if the equality holds when $U = \mathbb{R}^N$. The first (second) property is typical of lsc (usc) functions and, even when $U$ is an ordinary open subset, there seems to be no record that they both hold for all quasiconvex functions.

This property ensures that the pointwise extrema of $f$ on any nonempty density open subset can be arbitrarily closely approximated by values of $f$ achieved on “large” subsets, which may be of relevance in a variety of issues. To support this claim, we use it to characterize the common points of continuity, or approximate continuity, of two quasiconvex functions that coincide away from a set of measure zero.