A Note on Conjectures of F. Galvin and R. Rado
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Abstract. In 1968, Galvin conjectured that an uncountable poset $P$ is the union of countably many chains if and only if this is true for every subposet $Q \subseteq P$ with size $\aleph_1$. In 1981, Rado formulated a similar conjecture that an uncountable interval graph $G$ is countably chromatic if and only if this is true for every induced subgraph $H \subseteq G$ with size $\aleph_1$. Todorčević has shown that Rado’s Conjecture is consistent relative to the existence of a supercompact cardinal, while the consistency of Galvin’s Conjecture remains open. In this paper, we survey and collect a variety of results related to these two conjectures. We also show that the extension of Rado’s conjecture to the class of all chordal graphs is relatively consistent with the existence of a supercompact cardinal.