A Cohomological Property of $\pi$-invariant Elements
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Abstract. Let $A$ be a Banach algebra and $\pi : A \to \mathcal{L}(H)$ be a continuous representation of $A$ on a separable Hilbert space $H$ with $\dim H = m$. Let $\pi_{ij}$ be the coordinate functions of $\pi$ with respect to an orthonormal basis and suppose that for each $1 \leq j \leq m$, $C_j = \sum_{i=1}^{m} \|\pi_{ij}\|_{A^*} < \infty$ and $\sup_j C_j < \infty$. Under these conditions, we call an element $\Phi \in l^\infty(m, A^{**})$ left $\pi$-invariant if $a \cdot \Phi = ^t\pi(a)\Phi$ for all $a \in A$. In this paper we prove a link between the existence of left $\pi$-invariant elements and the vanishing of certain Hochschild cohomology groups of $A$. Our results extend an earlier result by Lau on $F$-algebras and recent results of Kaniuth–Lau–Pym and the second named author in the special case that $\pi : A \to \mathbb{C}$ is a non-zero character on $A$. 