Transcendental Solutions of a Class of Minimal Functional Equations

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Abstract. We prove a result concerning power series \( f(z) \in \mathbb{C}[z] \) satisfying a functional equation of the form

\[
f(z^n) = \sum_{k=1}^{n} \frac{A_k(z)}{B_k(z)} f(z)^k,
\]

where \( A_k(z), B_k(z) \in \mathbb{C}[z] \). In particular, we show that if \( f(z) \) satisfies a minimal functional equation of the above form with \( n \geq 2 \), then \( f(z) \) is necessarily transcendental. Towards a more complete classification, the case \( n = 1 \) is also considered.

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