The Milnor–Stasheff Filtration on Spaces
and Generalized Cyclic Maps

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Abstract. The concept of $C_k$-spaces is introduced, situated at an intermediate stage between $H$-spaces and $T$-spaces. The $C_k$-space corresponds to the $k$-th Milnor–Stasheff filtration on spaces. It is proved that a space $X$ is a $C_k$-space if and only if the Gottlieb set $G(Z, X) = [Z, X]$ for any space $Z$ with $\text{cat} Z \leq k$, which generalizes the fact that $X$ is a $T$-space if and only if $G(\Sigma B, X) = [\Sigma B, X]$ for any space $B$. Some results on the $C_k$-space are generalized to the $C^f_k$-space for a map $f : A \to X$. Projective spaces, lens spaces and spaces with a few cells are studied as examples of $C_k$-spaces, and non-$C_k$-spaces.

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