Almost Everywhere Convergence of Convolution Measures

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Abstract. Let \((X, \mathcal{B}, m, \tau)\) be a dynamical system with \((X, \mathcal{B}, m)\) a probability space and \(\tau\) an invertible, measure preserving transformation. This paper deals with the almost everywhere convergence in \(L^1(X)\) of a sequence of operators of weighted averages. Almost everywhere convergence follows once we obtain an appropriate maximal estimate and once we provide a dense class where convergence holds almost everywhere. The weights are given by convolution products of members of a sequence of probability measures \(\{\nu_i\}\) defined on \(\mathbb{Z}\). We then exhibit cases of such averages where convergence fails.

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