The Group \( \text{Aut}(\mu) \) is Roelcke Precompact

Eli Glasner

Abstract. Following a similar result of Uspenskij on the unitary group of a separable Hilbert space, we show that, with respect to the lower (or Roelcke) uniform structure, the Polish group \( G = \text{Aut}(\mu) \) of automorphisms of an atomless standard Borel probability space \((X, \mu)\) is precompact. We identify the corresponding compactification as the space of Markov operators on \( L_2(\mu) \) and deduce that the algebra of right and left uniformly continuous functions, the algebra of weakly almost periodic functions, and the algebra of Hilbert functions on \( G \), i.e., functions on \( G \) arising from unitary representations, all coincide. Again following Uspenskij, we also conclude that \( G \) is totally minimal.

Department of Mathematics, Tel Aviv University, Ramat Aviv, Israel
e-mail: glasner@math.tau.ac.il

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