Convergence Rates of Cascade Algorithms with Infinitely Supported Masks

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Abstract. We investigate the solutions of refinement equations of the form

$$\phi(x) = \sum_{\alpha \in \mathbb{Z}^s} a(\alpha) \phi(Mx - \alpha),$$

where the function $\phi$ is in $L^p(\mathbb{R}^s)(1 \leq p \leq \infty)$, $a$ is an infinitely supported sequence on $\mathbb{Z}^s$ called a refinement mask, and $M$ is an $s \times s$ integer matrix such that $\lim_{n \to \infty} M^{-n} = 0$. Associated with the mask $a$ and $M$ is a linear operator $Q_{a,M}$ defined on $L^p(\mathbb{R}^s)$ by

$$Q_{a,M}\phi_0 := \sum_{\alpha \in \mathbb{Z}^s} a(\alpha)\phi(M \cdot - \alpha).$$

Main results of this paper are related to the convergence rates of $(Q_{a,M}^n\phi_0)_{n=0,1,2,...}$ in $L^p(\mathbb{R}^s)$ with mask $a$ being infinitely supported. It is proved that under some appropriate conditions on the initial function $\phi_0$, $Q_{a,M}^n\phi_0$ converges in $L^p(\mathbb{R}^s)$ with an exponential rate.

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