Atomic Decomposition and Boundedness of Operators on Weighted Hardy Spaces

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Abstract. In this article, we establish a new atomic decomposition for \( f \in L^2_w \cap H^p_w \), where the decomposition converges in \( L^2_w \)-norm rather than in the distribution sense. As applications of this decomposition, assuming that \( T \) is a linear operator bounded on \( L^p_w \) and \( 0 < p \leq 1 \), we obtain (i) if \( T \) is uniformly bounded in \( L^p_w \)-norm for all \( w \)-\( p \)-atoms, then \( T \) can be extended to be bounded from \( H^p_w \) to \( L^p_w \); (ii) if \( T \) is uniformly bounded in \( H^p_w \)-norm for all \( w \)-\( p \)-atoms, then \( T \) can be extended to be bounded on \( H^p_w \); (iii) if \( T \) is bounded on \( H^p_w \), then \( T \) can be extended to be bounded from \( H^p_w \) to \( L^p_w \).

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