From Matrix to Operator Inequalities

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Abstract. We generalize Löwner’s method for proving that matrix monotone functions are operator monotone. The relation $x \leq y$ on bounded operators is our model for a definition of $C^*$-relations being residually finite dimensional.

Our main result is a meta-theorem about theorems involving relations on bounded operators. If we can show there are residually finite dimensional relations involved and verify a technical condition, then such a theorem will follow from its restriction to matrices.

Applications are shown regarding norms of exponentials, the norms of commutators, and "positive" noncommutative $*$-polynomials.

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