Involution Fixing $F^n \cup \{\text{Indecomposable}\}$

Pedro L. Q. Pergher

Abstract. Let $M^m$ be an $m$-dimensional, closed and smooth manifold, equipped with a smooth involution $T : M^m \to M^m$ whose fixed point set has the form $F^n \cup F^j$, where $F^n$ and $F^j$ are submanifolds with dimensions $n$ and $j$, $F^j$ is indecomposable and $n > j$. Write $n - j = 2q$, where $q \geq 1$ is odd and $p \geq 0$, and set $m(n - j) = 2n + p - q + 1$ if $p \leq q + 1$ and $m(n - j) = 2n + 2^{p+q}$ if $p \geq q$. In this paper we show that $m \leq m(n - j) + 2j + 1$. Further, we show that this bound is almost best possible, by exhibiting examples $(M^{m(n-j)2j}, T)$ where the fixed point set of $T$ has the form $F^n \cup F^j$ described above, for every $2 \leq j < n$ and $j$ not of the form $2t - 1$ (for $j = 0$ and 2, it has been previously shown that $m(n - j) + 2j$ is the best possible bound). The existence of these bounds is guaranteed by the famous $5/2$-theorem of J. Boardman, which establishes that under the above hypotheses $m \leq \frac{5}{2}n$.

Departamento de Matemática, Universidade Federal de São Carlos, Caixa Postal 676, São Carlos, SP 13565-905, Brazil

e-mail: pergher@dm.ufscar.br

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