Artinian and Non-Artinian Local Cohomology Modules

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Abstract. Let $M$ be a finite module over a commutative noetherian ring $R$. For ideals $\alpha$ and $\beta$ of $R$, the relations between cohomological dimensions of $M$ with respect to $\alpha$, $\beta$, $\alpha \cap \beta$ and $\alpha + \beta$ are studied. When $R$ is local, it is shown that $M$ is generalized Cohen–Macaulay if there exists an ideal $\alpha$ such that all local cohomology modules of $M$ with respect to $\alpha$ have finite lengths. Also, when $r$ is an integer such that $0 \leq r < \dim_R(M)$, any maximal element $\alpha$ of the non-empty set of ideals $\{ \alpha : H^i_\alpha(M) \text{ is not artinian for some } i, i \geq r \}$ is a prime ideal, and all Bass numbers of $H^i_\alpha(M)$ are finite for all $i \geq r$.

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Received by the editors September 27, 2008; revised November 17, 2008.
Published electronically March 11, 2011.
M. T. Dibaei was supported by a grant from IPM No. 87130117.
AMS subject classification: 13D45, 13E10.
Keywords: local cohomology modules, cohomological dimensions, Bass numbers.