An Extension of Craig’s Family of Lattices

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Abstract. Let \( p \) be a prime, and let \( \zeta_p \) be a primitive \( p \)-th root of unity. The lattices in Craig’s family are \((p − 1)\)-dimensional and are geometrical representations of the integral \( \mathbb{Z}[\zeta_p] \)-ideals \( \langle 1 − \zeta_p^i \rangle \), where \( i \) is a positive integer. This lattice construction technique is a powerful one. Indeed, in dimensions \( p − 1 \) where \( 149 \leq p \leq 3001 \), Craig’s lattices are the densest packings known. Motivated by this, we construct \((p − 1)(q − 1)\)-dimensional lattices from the integral \( \mathbb{Z}[\zeta_{pq}] \)-ideals \( \langle 1 − \zeta_p^i \rangle \langle 1 − \zeta_q^j \rangle \), where \( p \) and \( q \) are distinct primes and \( i \) and \( j \) are positive integers. In terms of sphere-packing density, the new lattices and those in Craig’s family have the same asymptotic behavior. In conclusion, Craig’s family is greatly extended while preserving its sphere-packing properties.