2-Local Isometries on Spaces of Lipschitz Functions

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Abstract. Let \((X, d)\) be a metric space, and let \(\text{Lip}(X)\) denote the Banach space of all scalar-valued bounded Lipschitz functions \(f\) on \(X\) endowed with one of the natural norms

\[
\|f\| = \max\{\|f\|_\infty, L(f)\} \quad \text{or} \quad \|f\| = \|f\|_\infty + L(f),
\]

where \(L(f)\) is the Lipschitz constant of \(f\). It is said that the isometry group of \(\text{Lip}(X)\) is canonical if every surjective linear isometry of \(\text{Lip}(X)\) is induced by a surjective isometry of \(X\). In this paper we prove that if \(X\) is bounded separable and the isometry group of \(\text{Lip}(X)\) is canonical, then every 2-local isometry of \(\text{Lip}(X)\) is a surjective linear isometry. Furthermore, we give a complete description of all 2-local isometries of \(\text{Lip}(X)\) when \(X\) is bounded.

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