Abstract. In this paper, we apply the saddle-point method in conjunction with the theory of the Norlund–Rice integrals to derive precise asymptotic formula for the generalized Li coefficients established by Omar and Mazhouda. Actually, for any function $F$ in the Selberg class $S$ and under the Generalized Riemann Hypothesis, we have

$$\lambda_F(n) = \frac{d_F}{2} n \log n + c_F n + O(\sqrt{n} \log n),$$

with

$$c_F = \frac{d_F}{2} (\gamma - 1) + \frac{1}{2} \log(\lambda Q_2), \quad \lambda = \prod_{j=1}^{r} \lambda_j^{2\lambda_j},$$

where $\gamma$ is the Euler’s constant and the notation is as below.

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