Maximal Sublattices of Finite Distributive Lattices. III: A Conjecture from the 1984 Banff Conference on Graphs and Order

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Abstract. Let $L$ be a finite distributive lattice. Let $\text{Sub}_0(L)$ be the lattice
\[
\{S \mid S \text{ is a sublattice of } L\} \cup \{\emptyset\}
\]
and let $\ell_*[\text{Sub}_0(L)]$ be the length of the shortest maximal chain in $\text{Sub}_0(L)$. It is proved that if $K$ and $L$ are non-trivial finite distributive lattices, then
\[
\ell_*[\text{Sub}_0(K \times L)] = \ell_*[\text{Sub}_0(K)] + \ell_*[\text{Sub}_0(L)].
\]
A conjecture from the 1984 Banff Conference on Graphs and Order is thus proved.

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