On the Dichotomy of the Evolution Families: A Discrete-Argument Approach

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Abstract. We establish a discrete-time criteria guaranteeing the existence of an exponential dichotomy in the continuous-time behavior of an abstract evolution family. We prove that an evolution family $\mathcal{U} = \{U(t,s)\}_{t \geq s \geq 0}$ acting on a Banach space $X$ is uniformly exponentially dichotomic (with respect to its continuous-time behavior) if and only if the corresponding difference equation with the inhomogeneous term from a vector-valued Orlicz sequence space $l^\Phi(N,X)$ admits a solution in the same $l^\Phi(N,X)$. The technique of proof effectively eliminates the continuity hypothesis on the evolution family (i.e., we do not assume that $U(\cdot,s)x$ or $U(t,\cdot)x$ is continuous on $[s, \infty)$, and respectively $[0,t]$). Thus, some known results given by Coffman and Schaffer, Perron, and Ta Li are extended.

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