Measurements and $G_δ$-Subsets of Domains

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Abstract. In this paper we study domains, Scott domains, and the existence of measurements. We use a space created by D. K. Burke to show that there is a Scott domain $P$ for which $\text{max}(P)$ is a $G_δ$-subset of $P$ and yet no measurement $\mu$ on $P$ has $\ker(\mu) = \text{max}(P)$. We also correct a mistake in the literature asserting that $[0, \omega_1)$ is a space of this type. We show that if $P$ is a Scott domain and $X \subseteq \text{max}(P)$ is a $G_δ$-subset of $P$, then $X$ has a $G_δ$-diagonal and is weakly developable. We show that if $X \subseteq \text{max}(P)$ is a $G_δ$-subset of $P$, where $P$ is a domain but perhaps not a Scott domain, then $X$ is domain-representable, first-countable, and is the union of dense, completely metrizable subspaces. We also show that there is a domain $P$ such that $\text{max}(P)$ is the usual space of countable ordinals and is a $G_δ$-subset of $P$ in the Scott topology. Finally we show that the kernel of a measurement on a Scott domain can consistently be a normal, separable, non-metrizable Moore space.

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