Numerical Semigroups That Are Not Intersections of $d$-Squashed Semigroups

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Abstract. We say that a numerical semigroup is $d$-squashed if it can be written in the form

$$S = \frac{1}{N}(a_1, \ldots, a_d) \cap \mathbb{Z}$$

for $N, a_1, \ldots, a_d$ positive integers with $\gcd(a_1, \ldots, a_d) = 1$. Rosales and Urbano have shown that a numerical semigroup is 2-squashed if and only if it is proportionally modular.

Recent works by Rosales et al. give a concrete example of a numerical semigroup that cannot be written as an intersection of 2-squashed semigroups. We will show the existence of infinitely many numerical semigroups that cannot be written as an intersection of 2-squashed semigroups. We also will prove the same result for 3-squashed semigroups. We conjecture that there are numerical semigroups that cannot be written as the intersection of $d$-squashed semigroups for any fixed $d$, and we prove some partial results towards this conjecture.

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